Contents

Chapter 1
Asymptotic methods for random tessellations

Pierre Calka

Abstract In this chapter, we are interested in two classical examples of random tessellations which are the Poisson hyperplane tessellation and Poisson-Voronoi tessellation. The first section introduces the main definitions, the application of an ergodic theorem and the construction of the so-called typical cell as the natural object for a statistical study of the tessellation. We investigate a few asymptotic properties of the typical cell by estimating the distribution tails of some of its geometric characteristics (inradius, volume, fundamental frequency). In the second section, we focus on the particular situation where the inradius of the typical cell is large. We start with precise distributional properties of the circumscribed radius that we use afterwards to provide quantitative information about the closeness of the cell to a ball. We conclude with limit theorems for the number of hyperfaces when the inradius goes to infinity.

1.1 Random tessellations - Distribution estimates

This section is devoted to the introduction of the main notions related to random tessellations and to some examples of distribution tail estimates. In the first subsection, we define the two main examples of random tessellations, namely the *Poisson hyperplane tessellation* and the *Poisson-Voronoi tessellation*. The next subsection is restricted to the stationary tessellations for which it is possible to construct a statistical object called *typical cell* $\mathcal{C}$ via several techniques (ergodicity, Palm measures, explicit realizations). Having isolated the cell $\mathcal{C}$, i.e. a random polyhedron which represents a cell „picked at random" in the whole tessellation, we can investigate its

Pierre Calka

Laboratoire de Mathématiques Raphaël Salem, Université de Rouen, Avenue de l'Université, BP.12 Technopôle du Madrillet F76801 Saint-Étienne-du-Rouvray France, e-mail: pierre.calka@univ-rouen.fr

geometric characteristics. In the last subsection, we present techniques for estimating their distribution tails.

This section does not intend to provide the most general definitions and results. It rather aims at emphasizing some basic examples. Quite often, we shall consider the particular case of the plane. A more exhaustive study of random tessellations can be found in the books [**?**], [**?**], [**?**] and [**?**] as well as the surveys [**?**] and [**?**].

1.1.1 Definitions

Definition 1.1. (Convex tessellation) A *convex tessellation* is a locally finite collection $\{\Xi_n\}_{n\in\mathbb{N}}$ of convex polyhedra of $\mathbb{R}^d$ such that $\cup_{n\in\mathbb{N}}\Xi_n = \mathbb{R}^d$ and Ξ_n and Ξ_m have disjoint interiors if $n \neq m$. Each Ξ_n is called a *cell* of the tessellation.

The set $\mathbb{T}$ of convex tessellations is endowed with the σ-algebra generated by the sets

$$\{\{\Xi_n\}_{n\in\mathbb{N}} : [\cup_{n\in\mathbb{N}}\partial\Xi_n]\cap K = \emptyset\}$$

where K is any compact set of $\mathbb{R}^d$.

Definition 1.2. (Random convex tessellation) A *random convex tessellation* is a random variable with values in $\mathbb{T}$.

Remark 1.1. We can equivalently identify a tessellation $\{\Xi_n\}_{n\in\mathbb{N}}$ with its skeleton $\cup_{n\in\mathbb{N}}\partial\Xi_n$ which is a random closed set of $\mathbb{R}^d$.

Definition 1.3. (Stationarity, isotropy) A random convex tessellation is *stationary* (resp. *isotropic*) if its skeleton is a translation-invariant (resp. rotation-invariant) random closed set.

We describe below the two classical constructions of random convex tessellations, namely the hyperplane tessellation and the Voronoi tessellation. In the rest of the section, we shall only consider these two particular examples even though many more can be found in the literature (Laguerre tessellations [**?**], iterated tessellations [**?**], Johnson-Mehl tessellations [**?**], crack STIT tessellations [**?**], etc.).

Definition 1.4. (Hyperplane tessellation) Let X be a point process which does not contain the origin almost surely. For every $x \in X$, we define its *polar hyperplane* as $H_x = \{y \in \mathbb{R}^d : \langle y - x, x \rangle = 0\}$. The associated *hyperplane tessellation* is the set of the closure of all connected components of $\mathbb{R}^d \setminus \cup_{x\in X} H_x$.

We focus on the particular case where X is a Poisson point process. The next proposition provides criteria for stationarity and isotropy.

Proposition 1.1. (Stationarity of Poisson hyperplane tessellations) *Let X be a Poisson point process of intensity measure* μ.

The associated hyperplane tessellation is stationary iff μ *can be written in function of spherical coordinates* $(u,t) \in \mathbb{S}^{d-1} \times \mathbb{R}_+$ *as*

$$\mu(du, dt) = \lambda \, dt \, \varphi(du) \tag{1.1}$$

where φ is a probability measure on $\mathbb{S}^{d-1}$.

It is additionally isotropic iff φ is the uniform measure σ_{d-1} on $\mathbb{S}^{d-1}$.

A so-called *Poisson hyperplane tessellation* (*Poisson line tessellation* in dimension two) is a hyperplane tessellation generated by a Poisson point process but it is quite often implied in the literature that it is also stationary and isotropic. Up to rescaling, we will assume in the rest of the chapter that its intensity λ is equal to one.

Exercise 1.1. Verify that a stationary and isotropic Poisson hyperplane tessellation satisfies the following property with probability one: for $0 \le k \le d$, each k-dimensional face of a cell is the intersection of exactly $(d-k)$ hyperplanes H_x, $x \in X$, and is included in exactly 2^{d-k} cells.

This tessellation has been introduced for studying trajectories in bubble chambers by S. A. Goudsmit in [**?**] in 1945. It has been used in numerous applied works since then. For instance, R. E. Miles describes it as a possible model for the fibrous structure of sheets of paper [**?**, **?**].

Definition 1.5. (Voronoi tessellation) Let X be a point process. For every $x \in X$, we define the *cell associated with x* as

$$C(x|X) = \{y \in \mathbb{R}^d : \|y - x\| \le \|y - x'\| \; \forall x' \in X, x' \neq x\}.$$

The associated *Voronoi tessellation* is the set $\{C(x|X)\}_{x \in X}$.

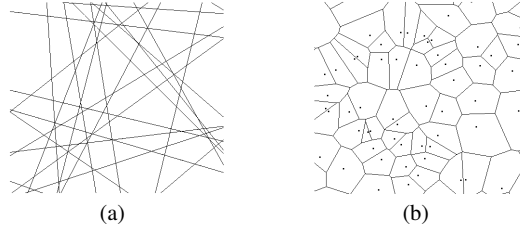(a)   (b)

Fig. 1.1 Realizations of the isotropic and stationary Poisson line tessellation (a) and the planar stationary Poisson-Voronoi tessellation (b) in the unit square

Proposition 1.2. (Stationarity of Voronoi tessellations) *The Voronoi tessellation associated with a point process X is stationary if and only if X is stationary.*

A so-called *Poisson-Voronoi tessellation* is a Voronoi tessellation generated by a homogeneous Poisson point process. Up to rescaling, we will assume in the rest of the chapter that its intensity is equal to one.

Exercise 1.2. Show that a Poisson-Voronoi tessellation is *normal* with probability one, i.e. every k-dimensional face of a cell, $0 \leq k \leq d$, is included in exactly $d - k + 1$ cells.

This tessellation has been introduced in a deterministic context by R. Descartes in 1644 as a description of the structure of the universe (see also the more recent work [**?**]). It has been developed since then for many applications, e.g. in telecommunications [**?**, **?**], image analysis [**?**] and molecular biology [**?**].

We face the whole population of cells in a random tessellation. How to study them? One can provide two possible answers:

1. either you isolate one particular cell,
2. or you try conversely to do a statistical study over all the cells by taking means.

An easy way to fix a cell consists in considering the one containing the origin.

Definition 1.6. (Zero-cell) If $o \notin \cup_{i \in \mathbb{N}} \partial C_i$ a.s., then the *zero-cell* (denoted by C_0) is the cell containing the origin. In the case of an isotropic and stationary Poisson hyperplane tessellation, it is called the *Crofton cell*.

The second point above will be developed in the next section. It is intuitively clear that it will be possible to show the convergence of means over all cells only if the tessellation is translation-invariant.

1.1.2 Empirical means and typical cell

This section is restricted to the stationary Poisson-Voronoi and Poisson hyperplane tessellations. We aim at taking means of certain characteristics over all the cells of the tessellation. But of course, we have to restrict the mean to a finite number of these cells due to technical reasons. A natural idea is to consider those contained in or intersecting a fixed window, e.g. the ball $B_R(o)$, then take the limit when the size of the window goes to infinity. Such an argument requires the use of an ergodic theorem and the first part of the section will be devoted to prepare and show an ergodic result specialized to our set-up. In the second part of the section, we use it to define the notion of the typical cell and we investigate several equivalent ways of defining it.

1.1.2.1 Ergodic theorem for tessellations

The first step is to realize the measurable space $(\Omega, \mathcal{F})$ as $(\mathsf{N}, \mathcal{N})$ where N is the set of locally finite sets of $\mathbb{R}^d$ and $\mathcal{N}$ is the σ-algebra generated by the functions $\#(\cdot \cap A)$, where A is any bounded Borel set. We define the *shift* $S^a : \mathsf{N} \longrightarrow \mathsf{N}$ as the operation over the points needed to translate the tessellation by a vector $a \in \mathbb{R}^d$. In other words, for every locally finite set $\{x_n\}_{n \in \mathbb{N}}$, the underlying tessellation

generated by $S^a(\{x_n\}_{n\in\mathbb{N}})$ is the translate by a of the initial tessellation generated by $\{x_n\}_{n\in\mathbb{N}}$.

Proposition 1.3. (Explicit shifts) *For a Voronoi tessellation, S^a is the function which associates to every locally finite set $\{x_n\}_{n\in\mathbb{N}} \in \mathbb{N}$ the set $\{x_n + a\}_{n\in\mathbb{N}}$.*

For a hyperplane tessellation, S^a is the function which associates to every locally finite set $\{x_n\}_{n\in\mathbb{N}} \in \mathbb{N}$ (which does not contain the origin) the set $\{x_n + \langle x_n/\|x_n\|, a\rangle x_n/\|x_n\|\}_{n\in\mathbb{N}}$.

Proof. In the Voronoi case, the translation of the skeleton is equivalent with the translation of the nuclei which generate the tessellation.

In the case of a hyperplane tessellation, the translation of a fixed hyperplane preserves its orientation but modifies the distance from the origin. To prove the proposition, it suffices to notice that a polar hyperplane H_x is sent by a translation of vector a to H_y with $y = x + \langle u, a\rangle u$ where $u = \frac{x}{\|x\|}$. $\qquad\square$

Proposition 1.4. (Ergodicity of the shifts) *In both cases, S^a preserves the measure* **P** *(i.e. the distribution of the Poisson point process) and is ergodic.*

Sketch of proof. Saying that S^a preserves the measure is another way of expressing the stationarity of the tessellation.

To show ergodicity, it is sufficient to prove that $\{S^a : a \in \mathbb{R}^d\}$ is mixing (cf. Definition 4.6), i.e. that for any bounded Borel sets A, B and $k, l \in \mathbb{N}$, we have as $|a| \to \infty$

$$\mathbf{P}(\#(X \cap A) = k; \#(S^a(X) \cap B) = l) \to \mathbf{P}(\#(X \cap A) = k)\mathbf{P}(\#(X \cap B) = l). \quad (1.2)$$

In the Voronoi case, for $|a|$ large enough, the two events $\{\#(\Phi \cap A) = k\}$ and $\{\#(S^a(\Phi) \cap B) = l\}$ are independent since $A \cap (B - a) = \emptyset$. Consequently, the two sides of (**??**) are equal.

In the hyperplane case, the same occurs as soon as B is included in a set

$$\{x \in \mathbb{R}^d : |\langle x, a\rangle| \geq \varepsilon\|x\|\|a\|\}$$

for some $\varepsilon > 0$. Otherwise, we approximate B with a sequence of Borel subsets which satisfy this condition. $\qquad\square$

In the next theorem, the main application of ergodicity for tessellations is derived.

Theorem 1.1. (Ergodic theorem for tessellations) *Let N_R be the number of cells which are included in the ball $B_R(o)$. Let $h : \mathcal{K}_{\text{conv}} \to \mathbb{R}$ be a measurable, bounded and translation-invariant function over the set $\mathcal{K}_{\text{conv}}$ of convex and compact sets of $\mathbb{R}^d$. Then almost surely,*

$$\lim_{R\to\infty} \frac{1}{N_R} \sum_{C \subset B_R(o)} h(C) = \frac{1}{\mathbf{E}(v_d(C_0)^{-1})} \mathbf{E}\left(\frac{h(C_0)}{v_d(C_0)}\right). \quad (1.3)$$

Proof. The proof is done in three steps: use of Wiener's continuous ergodic theorem, then rewriting the mean of h over cells included in $B_R(o)$ as the sum of an

integral and the rest, finally proving that the rest is negligible.

Step 1. The main ingredient is Wiener's ergodic theorem applied to the ergodic shifts $\{S^x : x \in \mathbb{R}^d\}$. We have almost surely

$$\lim_{R \to \infty} \frac{1}{v_d(B_R(o))} \int_{B_R(o)} \frac{h(C_0(S^{-x}\omega))}{v_d(C_0(S^{-x}\omega))} \, dx = \mathbf{E}\left(\frac{h(C_0)}{v_d(C_0)}\right).$$

This can be roughly interpreted by saying that the mean in space (in the left-hand side) for a fixed sample ω is asymptotically close to the mean with respect to the probability law $\mathbf{P}$.

Step 2. We have for almost every $\omega \in \Omega$ that

$$\frac{1}{v_d(B_R(o))} \int_{B_R(o)} \frac{h(C_0(S^{-x}\omega))}{v_d(C_0(S^{-x}\omega))} \, dx = \frac{1}{v_d(B_R(o))} \sum_{C \subset B_R(o)} h(C) + \mathrm{Rest}(R) \quad (1.4)$$

where

$$\mathrm{Rest}(R) = \frac{1}{v_d(B_R(o))} \sum_{C : C \cap \partial B_R(o) \neq \emptyset} \frac{v_d(C \cap B_R(o))}{v_d(C)} h(C).$$

In particular, if we define N_R' as the number of cells which intersect the boundary of the ball $B_R(o)$, then there is a positive constant K depending only on h such that

$$|\mathrm{Rest}(R)| \leq K \frac{N_R'}{v_d(B_R(o))}.$$

We observe that in order to get (**??**), it is enough to prove that the rest goes to 0. Indeed, when $h \equiv 1$, the equality (**??**) will provide that

$$\frac{N_R}{v_d(B_R(o))} = \frac{1}{v_d(B_R(o))} \int_{B_R(o)} \frac{1}{v_d(C_0(S^{-x}\omega))} \, dx - \mathrm{Rest}(R) \to \mathbf{E}(v_d(C_0)^{-1}).$$

Step 3. We have to show that $\mathrm{Rest}(R)$ goes to 0, that is what R. Cowan calls the insignificance of edge effects [**?**, **?**]. In the sequel, we use his argument to show it and for sake of simplicity, we only consider the particular case of the two-dimensional Voronoi tessellation. Nevertheless, the method can be extended to any dimension by showing by induction that the number of k-faces hitting the boundary of the ball is negligible for every $0 \leq k \leq d$.

Here, in the two-dimensional case, let us fix $\varepsilon > 0$ and consider $R > \varepsilon$. Let us denote by V_R the number of vertices of the tessellation in $B_R(o)$ and by L_R the sum of the edge lengths inside $B_R(o)$. A direct use of Wiener's theorem applied to the functionals V_ε and L_ε on the sets $B_{R-\varepsilon}(o)$ and $B_{R+\varepsilon}(o)$ shows that the quantities $V_R/v_d(B_R(o))$ and $L_R/v_d(B_R(o))$ tend almost surely to constants.

We recall that a Voronoi tessellation is *normal* which means in particular that a fixed edge (resp. vertex) is contained in exactly two (resp. three) cells.

If a cell intersects the boundary of $B_R(o)$, then we are in one of the three following cases:

1. no edge of the cell intersects $B_{R+\varepsilon}(o) \setminus B_R(o)$,
2. some edges but no vertex of the cell intersect $B_{R+\varepsilon}(o) \setminus B_R(o)$,
3. at least one vertex of the cell is in $B_{R+\varepsilon}(o) \setminus B_R(o)$.

The first case is satisfied by at most one cell, the second case by at most $\frac{2}{\varepsilon}(L_{R+\varepsilon} - L_R)$ cells and the last one by at most $3(S_{R+\varepsilon} - S_R)$. Consequently, we get when $R \to \infty$

$$\frac{N'_R}{V_2(B_R(o))} \leq \frac{1}{V_2(B_R(o))} + 2\frac{L_{R+\varepsilon} - L_R}{V_2(B_R(o))} + 3\frac{V_{R+\varepsilon} - V_R}{V_2(B_R(o))} \longrightarrow 0,$$

which completes the proof. $\square$

Exercise 1.3. Show a similar result for a Johnson-Mehl tessellation (defined in [**?**]).

Remark 1.2. The statement of Theorem **??** still holds if the condition „h bounded" is replaced with $\mathbf{E}(|h(C_0)|^p) < \infty$ for a fixed $p > 1$ (see e.g. [**?**, Lemma 4]).

Remark 1.3. When using this ergodic theorem for tessellations in practice, it is needed to have also an associated central limit theorem. Such second-order results have been proved for some particular functionals in the Voronoi case [**?**] and for the Poisson line tessellation [**?**] in dimension two. Recently, a more general central limit result for hyperplane tessellations has been derived from the use of U-statistics in [**?**].

The limit in the convergence (**??**) suggests the next definition for the typical cell, i.e. a cell which represents an „average individual" from the whole population.

Definition 1.7. (Typical cell 1) The *typical cell* C is defined as a random variable with values in $\mathcal{K}_{\text{conv}}$ and such that for every translation-invariant measurable and bounded function $h : \mathcal{K}_{\text{conv}} \to \mathbb{R}$, we have

$$\mathbf{E}(h(C)) = \frac{1}{\mathbf{E}(v_d(C_0)^{-1})} \mathbf{E}\left(\frac{h(C_0)}{v_d(C_0)}\right).$$

Remark 1.4. Taking for h any indicator function of geometric events (e.g. {the cell is a triangle}, {the area of the cell is greater than 2}, etc.), we can define via the equality above the distribution of any geometric characteristic of the typical cell.

Remark 1.5. One should keep in mind that the typical cell C is not distributed as the zero-cell C_0. Indeed, the distribution of C has a density proportional to v_2^{-1} with respect to the distribution of C_0. In particular, since it has to contain the origin, C_0 is larger than C. This is a d-dimensional generalization of the famous bus paradox in renewal theory which states that at your arrival at a bus stop, the time interval between the last bus you missed and the first bus you'll get is actually bigger than the typical waiting time between two buses. Moreover, it has been proved in the case of a Poisson hyperplane tessellation that C and C_0 can be coupled in such a way that $C \subset C_0$ almost surely (see [**?**] and Proposition **??** below).

Looking at Definition **??**, we observe that it requires to know either the distribution of C_0 or the limit of the ergodic means in order to get the typical cell. The next definition is an alternative way of seeing the typical cell without the use of any convergence result. It is based on the theory of Palm measures [**?**, **?**]. For sake of simplicity, it is only written in the case of the Poisson-Voronoi tessellation but it can be extended easily to any stationary Poisson hyperplane tessellation.

Definition 1.8. (Typical cell 2 (Poisson-Voronoi tessellation)) The *typical cell C* is defined as a random variable with values in $\mathcal{K}_{\text{conv}}$ such that for every bounded and measurable function $h : \mathcal{K}_{\text{conv}} \to \mathbb{R}$ and every Borel set B with $0 < \nu_d(B) < \infty$, we have

$$\mathbf{E}(h(C)) = \frac{1}{\nu_d(B)} \mathbf{E}\left(\sum_{x \in B \cap X} h(C(x|X) - x) \right). \tag{1.5}$$

This second definition is still an intermediary and rather unsatisfying one but via the use of Slivnyak-Mecke formula for Poisson point processes (see Theorem **??**), it provides a way of realizing the typical cell C.

Exercise 1.4. Verify that the relation (**??**) does not depend on B.

Proposition 1.5. (Typical cell 3 (Poisson-Voronoi tessellation)) *The typical cell C is equal in distribution to the set $C(o) = C(o|X \cup \{o\})$, i.e. the Voronoi cell associated with a nucleus at the origin when this nucleus is added to the original Poisson point process.*
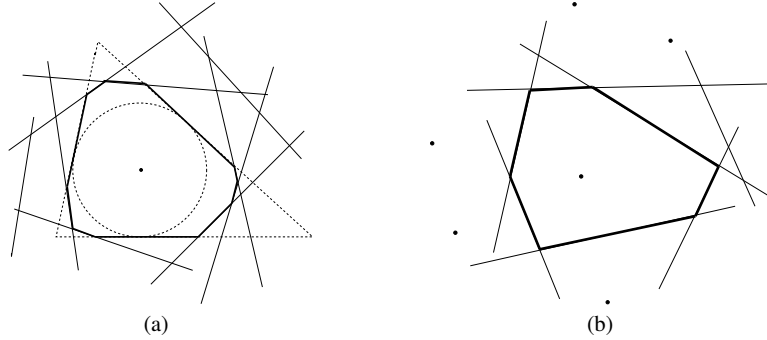

(a)   (b)

Fig. 1.2 Realizations of the typical cells of the stationary and isotropic Poisson line tessellation (a) and the homogeneous planar Poisson-Voronoi tessellation (b)

Remark 1.6. The cell $C(o)$ defined above is not a particular cell isolated from the original tessellation. It is a cell extracted from a different Voronoi tessellation but which has the right properties of a cell „picked at random" in the original tessellation. For any $x \in X$, we define the bisecting hyperplane of $[o, x]$ as the hyperplane

containing the midpoint $x/2$ and orthogonal to x. Since $C(o)$ is bounded by portions of bisecting hyperplanes of segments $[o,x]$, $x \in X$, we remark that $C(o)$ can be alternatively seen as the zero-cell of a (non-stationary) Poisson hyperplane tessellation associated with the homogeneous Poisson point process up to a multiplicative constant.

The Poisson-Voronoi tessellation is not the only tessellation such that the associated typical cell can be realized in an elementary way. There exist indeed several ways of realizing the typical cell of a stationary and isotropic Poisson hyperplane tessellation. We present below one of the possible constructions of $\mathcal{C}$, which offers the advantage of satisfying $\mathcal{C} \subset C_0$ almost surely. It is based on a work [**?**] which is is an extension in any dimension of an original idea in dimension two due to R. E. Miles [**?**].

Proposition 1.6. (Typical cell 3 (Poisson hyperplane tessellation)) *The radius R_m of the largest ball included in the typical cell $\mathcal{C}$ is an exponential variable of parameter the area of the unit sphere. Moreover, conditionally on R_m, the typical cell $\mathcal{C}$ is equal in distribution to the intersection of the two independent following random sets:*
(i) a random simplex with inscribed ball $B_{R_m}(o)$ such that the vector $(u_0, \ldots, u_d)$ of the $d+1$ normal unit-vectors is independent of R_m and has a density proportional to the volume of the simplex spanned by $u_0, \ldots, u_d$.
(ii) the zero-cell of an isotropic Poisson hyperplane tessellation outside $B_{R_m}(o)$ of intensity measure $\mu(du, dt) = \mathbf{1}(B_{R_m}(o)^c) \, dt \, d\sigma_{d-1}(u)$ (in spherical coordinates).

Exercise 1.5. When $d = 2$, let us denote by α, β, γ the angles between u_0 and u_1, u_1 and u_2, u_2 and u_0 respectively. Write the explicit density in (i) in function of α, β and γ.

Exercise 1.6. We replace each hyperplane H_x from a Poisson hyperplane tessellation by a ε-thickened hyperplane $H_x^{(\varepsilon)} = \{y \in \mathbb{R}^d : d(y, H_x) \leq \varepsilon\}$ where $\varepsilon > 0$ is fixed. Show that the distribution of the typical cell remains unchanged, i.e. is the same as for $\varepsilon = 0$.

We conclude this subsection with a very basic example of calculation of a mean value: it is well-known that in dimension two, the mean number of vertices of the typical cell is 4 for an isotropic Poisson line tessellation and 6 for a Poisson-Voronoi tessellation. We give below a small heuristic justification of this fact: for a Poisson line tessellation, each vertex is in 4 cells exactly and there are as many cells as vertices (each vertex is the highest point of exactly one cell) whereas in the Voronoi case, each vertex is in 3 cells exactly and there are twice more vertices than cells (each vertex is either the highest point or the lowest point of exactly one cell).

In the next subsection, we estimate the distribution tails of some geometric characteristics of the typical cell.

1.1.3 Examples of distribution tail estimates

Example 1.1. (Poisson hyperplane tessellation, Crofton cell, inradius) We consider a stationary and isotropic Poisson hyperplane tessellation, i.e. with an intensity measure equal to $\mu(du, dt) = dt\, d\sigma_{d-1}(u)$ in spherical coordinates (note that the constant λ appearing in (1.1) is chosen equal to one).

Let us denote by R_m the radius of the largest ball included in the Crofton cell and centered at the origin. Since it has to be centered at the origin, the ball $B_{R_m}(o)$ is not the real inball of the Crofton cell. Nevertheless, we shall omit that fact and call R_m the *inradius* in the rest of the chapter.

For every $r > 0$, we have

$$\mathbf{P}(R_m \geq r) = \mathbf{P}(X \cap B_r(o) = \emptyset)$$
$$= \exp\left\{-\int_0^r \int_{\mathbb{S}^{d-1}} dt\, d\sigma_{d-1}(u)\right\} = \mathrm{e}^{-d\kappa_d r}$$

where κ_d is the Lebesgue measure of the d-dimensional unit-ball. We can remark that it is the same distribution as the real inradius of the typical cell, i.e. the radius of the largest ball included in the typical cell with unfixed center (see [**?, ?**] and Proposition **??** above).

This result can be extended by showing that for every deterministic convex set C containing the origin, the probability $\mathbf{P}(C \subset C_0)$ is equal to $\exp\{-\frac{d}{2}\kappa_d V_1(C)\}$ where $V_1(C)$ is the mean width of C. In dimension two, the probability reduces to $\exp\{-P(C)\}$ where $P(C)$ is the perimeter of C.

Example 1.2. (Poisson-Voronoi tessellation, typical cell, inradius) We consider a homogeneous Poisson-Voronoi tessellation of intensity one in the rest of the section.

We realize its typical cell as $C(o) = C(o|X \cup \{o\})$ (see Proposition **??**). We consider the radius R_m of the largest ball included in $C(o)$ and centered at the origin. We call it *inradius* with the same abuse of language as in the previous example. The radius R_m is larger than r iff for every x, $\|x\| = r$, x is in $C(o)$, i.e. $B_r(x)$ does not intersect the Poisson point process X. In other words, for every $r > 0$, we have

$$\mathbf{P}(R_m \geq r) = \mathbf{P}\left(X \cap \bigcup_{x:\|x\|=r} B_r(x) = \emptyset\right)$$
$$= \mathbf{P}(X \cap B_{2r}(o) = \emptyset) = \mathrm{e}^{-2^d \kappa_d r^d}. \tag{1.6}$$

In general, for a deterministic convex set C containing the origin, we define the *Voronoi flower* $F(C) = \bigcup_{x \in C} B_{\|x\|}(x)$. We can show the following equality:

$$\mathbf{P}(C \subset C(o)) = \exp\{-v_d(F(C))\}.$$

Exercise 1.7. Verify that for any compact subset A of $\mathbb{R}^d$, the Voronoi flowers of A and of its convex hull coincide.
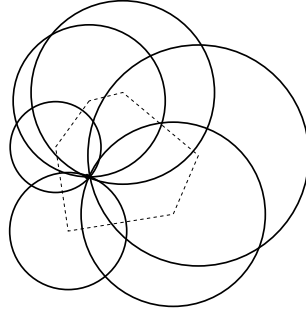
Fig. 1.3 Example of the Voronoi flower of a convex polygon

Example 1.3. (Poisson-Voronoi tessellation, typical cell, volume) The next proposition comes from a work due to E. N. Gilbert [**?**].

Proposition 1.7. (E. N. Gilbert, 1962) *For every $t > 0$, we have*

$$e^{-2^d t} \leq \mathbf{P}(v_d(\mathcal{C}) \geq t) \leq \frac{t-1}{e^{t-1}-1}.$$

Proof. *Lower bound:* it suffices to notice that $v_d(\mathcal{C}) \geq v_d(B_{R_m}(o))$ and apply (**??**). *Upper bound:* using Markov's inequality, we get for every $\alpha, t \geq 0$

$$\mathbf{P}(v_d(\mathcal{C}) \geq t) \leq (e^{\alpha t} - 1)^{-1}(\mathbf{E}(e^{\alpha v_d(\mathcal{C})}) - 1). \tag{1.7}$$

Let us consider now the quantity $f(\alpha) = \mathbf{E}\left(\int_{C(o)} e^{\alpha \kappa_d \|x\|^d} dx\right)$. On one hand, we can show by Fubini's theorem that for every $\alpha < 1$,

$$f(\alpha) = \int_{\mathbb{R}^d} e^{\alpha \kappa_d \|x\|^d} \mathbf{P}(x \in C(o)) dx = \int_{\mathbb{R}^d} e^{(\alpha-1)\kappa_d \|x\|^d} dx = \frac{1}{1-\alpha}. \tag{1.8}$$

On the other hand, when comparing $C(o)$ with the ball centered at the origin and of same volume, we use an isoperimetric inequality to get a lower bound for the same quantity:

$$f(\alpha) \geq \mathbf{E}\left(\int_{B_{(v_d(C(o))/\kappa_d)^{1/d}}(o)} e^{\alpha \kappa_d \|x\|^d} dx\right) = \frac{1}{\alpha}\mathbf{E}(e^{\alpha v_d(\mathcal{C})}). \tag{1.9}$$

Combining (**??**), (**??**) with (**??**), we obtain that for every $t > 0$,

$$\mathbf{P}(v_d(\mathcal{C}) \geq t) \leq (e^{\alpha t} - 1)^{-1}\frac{\alpha}{1-\alpha}$$

and it remains to optimize the inequality in α by taking $\alpha = \frac{t-1}{t}$. $\square$

Exercise 1.8. Show the isoperimetric inequality used above.

Remark 1.7. It has been proved since then (see Theorem **??** and [**?**]) that the lower bound provides the right logarithmic equivalent, i.e.

$$\lim_{t\to\infty} \frac{1}{t} \log \mathbf{P}(\nu_d(\mathcal{C}) \geq t) = -2^d.$$

In other words, distribution tails of the volumes of both the typical cell $\mathcal{C}$ and its inball have an analogous asymptotic behaviour. This is due to D. G. Kendall's conjecture (see the foreword of the book [**?**]) which was historically written for the two-dimensional Crofton cell. Indeed, it roughly states that cells with a large volume must be approximately spherical. After a first proof by I. N. Kovalenko [**?**], this conjecture has been rigorously reformulated and extended in many directions by D. Hug, M. Reitzner and R. Schneider (see Theorems **??** and **??** as well as [**?**, **?**]).

Example 1.4. (Poisson-Voronoi tessellation, typical cell, fundamental frequency in dimension two) This last more exotic example is motivated by the famous question due to Kac [**?**] back in 1966: „Can one hear the shape of a drum?“. In other words, let us consider the Laplacian equation on $C(o)$ with a Dirichlet condition on the boundary, that is

$$\begin{cases} \Delta f(x) = -\lambda f(x), & \text{if } x \in C(o), \\ f(x) = 0, & \text{if } x \in \partial C(o). \end{cases}$$

It has been proved that the eigenvalues satisfy

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \leq \cdots < \infty.$$

Is it possible to recover the shape of $C(o)$ by knowing only its spectrum? In particular, λ_1 is called the *fundamental frequency* of $C(o)$. It is a decreasing function of the convex set considered. When the volume of the domain is fixed, Faber-Krahn's inequality [**?**] says that it is minimal iff the domain is a ball. In such a case, we have $\lambda_1 = j_0^2/r^2$ where r is the radius of the ball and j_0 is the first positive zero of the Bessel function J_0 [**?**].

The next theorem which comes from a collaboration with A. Goldman [**?**] provides an estimate for the distribution function of λ_1 in the two-dimensional case.

Theorem 1.2. (Fundamental frequency of the typical Poisson-Voronoi cell) *Let μ_1 denote the fundamental frequency of the inball of $C(o)$. Then when $d = 2$, we have*

$$\lim_{t\to 0} t \cdot \log \mathbf{P}(\lambda_1 \leq t) = \lim_{t\to 0} t \cdot \log \mathbf{P}(\mu_1 \leq t) = -4\pi j_0^2.$$

Remark 1.8. The larger $C(o)$ is, the smaller is λ_1. When evaluating the probability of the event $\{\lambda_1 \leq t\}$ for small t, the contribution comes from the largest cells $C(o)$. Consequently, the fact that the distribution functions for small λ_1 and small μ_1 have roughly the same behaviour is a new contribution for justifying D. G. Kendall's conjecture.

Remark 1.9. An analogous result holds for the Crofton cell of a Poisson line tessellation in the plane [**?**].

Sketch of proof.

Step 1. By a Tauberian argument (see [**?**], Vol. 2, Ch. 13, pages 442-448), we only have to investigate the behaviour of the Laplace transform $\mathbf{E}(e^{-t\lambda_1})$ when t goes to infinity.

Step 2. We get a lower bound by using the monotonicity of the fundamental frequency ($\lambda_1 \le \mu_1$) and the explicit distribution of $\mu_1 = j_0^2/R_m^2$.

Step 3. In order to get an upper bound, we observe that almost surely

$$e^{-t\lambda_1} \le \varphi(t) = \sum_{n \ge 1} e^{-t\lambda_n}$$

where φ is called the *spectral function* of $C(o)$. It is known that the spectral function of a domain is connected to the probability that a two-dimensional Brownian bridge stays in that domain (see e.g. [**?**]). More precisely, we denote by W the trajectory of a standard two-dimensional Brownian bridge between 0 and 1 (i.e. a planar Brownian motion starting at 0 and conditioned on being at 0 at time 1) and independent from the point process. We have

$$\varphi(t) = \frac{1}{4\pi t} \int_{C(o)} \mathbf{P}^W(x + \sqrt{2t}W \subset C(o)) \, dx$$

where $\mathbf{P}^W$ denotes the probability with respect to the Brownian bridge W. We then take the expectation of the equality above with respect to the point process and we get the Laplace transform of the area of the Voronoi flower of the convex hull of W. We conclude by using results related to the geometry of the two-dimensional Brownian bridge [**?**]. □

Exercise 1.9. In the case of the Crofton cell, express $\varphi(t)$ in function of the Laplace transform of the mean width of W.

1.2 Asymptotic results for zero-cells with large inradius

In this section, we shall focus on the example of the Poisson-Voronoi typical cell, but the reader should keep in mind that the results discussed below can be extended to the Crofton cell and more generally to zero-cells of certain isotropic hyperplane tessellations (see [**?**, Sec. 3]). This section is devoted to the asymptotic behaviour of the typical cell, under the condition that it has large inradius. Though it may seem at first sight a very artificial and restrictive choice, we shall see that it falls in the more general context of D. G. Kendall's conjecture and that this particular conditioning allows us to obtain very precise estimations for the geometry of the cell.

In the first subsection, we are interested in the distribution tail of a particular geometric characteristic that we did not consider before, the so-called *circumscribed radius*. We deduce from the general techniques involved an asymptotic result for the joint distribution of the two radii. In the second subsection, we make the convergence of the cell to the spherical shape more precise by showing limit theorems

for some of its characteristics when the inradius goes to infinity. In this section, two fundamental models from stochastic geometry will be introduced as tools for understanding the geometry of the typical cell: random coverings of the circle/sphere and random polytopes generated as convex hulls of Poisson point processes in the ball.

1.2.1 Circumscribed radius

We consider a homogeneous Poisson-Voronoi tessellation of intensity one and we realize its typical cell as $C(o)$ according to Proposition **??**. With the same misuse of language as for the inradius, we define the *circumscribed radius R_M* of the typical cell $C(o)$ as the radius of the smallest ball containing $C(o)$ and centered at the origin. We first propose a basic way of estimating its distribution and we proceed with a more precise calculation through a technique based on coverings of the sphere which provides satisfying results essentially in dimension two.

1.2.1.1 Estimation of the distribution tail

For the sake of simplicity, the following argument is written only in dimension two and comes from an intermediary result of a work due to S. Foss and S. Zuyev [**?**]. We observe that R_M is larger than $r > 0$ iff there exists x, $\|x\| = r$, which is in $C(o)$, i.e. such that $B_{\|x\|}(x)$ does not intersect the Poisson point process X. Compared to the event $\{R_m \geq r\}$, the only difference is that „there exists x" is replacing „for every x" (see Example 2 of Section **??**).

In order to evaluate this probability, the idea is to discretize the boundary of the circle and consider a deterministic sequence of balls $B_{\|z_k\|}(z_k)$, $0 \leq k \leq (n-1)$, $n \in \mathbb{N} \setminus \{0\}$ with $z_k = r(\cos(2\pi k/n), \sin(2\pi k/n))$. We call the intersection of two consecutive such disks a petal. If $R_M \geq r$, then one of these n petals has to be empty. We can calculate the area of a petal and conclude that for every $r > 0$, we have

$$\mathbf{P}(R_M \geq r) \leq n \exp\{-r^2(\pi - \sin(2\pi/n) - (2\pi/n))\}. \tag{1.10}$$

In particular, when we look at the chord length in one fixed direction, i.e. the length l_u of the largest segment emanating from the origin in the direction u and contained in $C(o)$, we have directly for every $r > 0$,

$$\mathbf{P}(l_u \geq r) = \exp\{-\pi r^2\},$$

which seems to provide the same logarithmic equivalent as the estimation (**??**) when n goes to infinity. This statement will be reinforced in the next section.

1.2.1.2 Calculation and new estimation

This section and the next one present ideas and results contained in [**?, ?**]. The distribution of R_M can be calculated explicitly: let us recall that $C(o)$ can be seen as the intersection of half-spaces delimited by random bisecting hyperplanes and containing the origin. We then have $R_M \geq r$ ($r > 0$) if and only if the half-spaces do not cover the sphere $\partial B_r(o)$. Of course, only the hyperplanes which are at a distance less than r are necessary and their number is finite and Poisson distributed. The trace of a half-space on the sphere is a spherical cap with a (normalized) angular diameter α which is obviously less than $1/2$ and which has an explicit distribution. Indeed, α can be written in function of the distance L from the origin to the hyperplane via the formula $\alpha = \arccos(L/r)/\pi$. Moreover, the obtained spherical caps are independent. For any probability measure ν on $[0, 1/2]$ and $n \in \mathbb{N}$, we denote by $P(\nu, n)$ the probability to cover the unit-sphere with n i.i.d. isotropic spherical caps such that their normalized angular diameters are ν distributed. Following this reasoning, the next proposition connects the distribution tail of R_M with some covering probabilities $P(\nu, n)$.
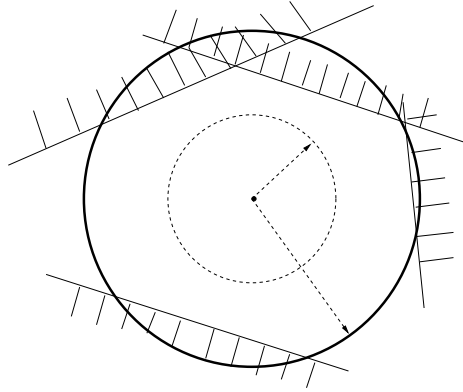

Fig. 1.4 Covering of the circle of radius r when $R_M \geq r$

Proposition 1.8. (Rewriting of the distribution tail of R_M) *For every $r > 0$, we have*

$$\mathbf{P}(R_M \geq r) = \mathrm{e}^{-2^d \kappa_d r^d} \sum_{n=0}^{\infty} \frac{(2^d \kappa_d r^d)^n}{n!} (1 - P(\nu, n)) \tag{1.11}$$

where ν is a probability measure on $[0, 1/2]$ with the density

$$f_\nu(\theta) = d\pi \sin(\pi\theta) \cos^{d-1}(\pi\theta), \quad \theta \in [0, 1/2].$$

Exercise 1.10. Verify the calculation of ν and do the same when the Poisson-Voronoi typical cell is replaced by the Crofton cell of a Poisson hyperplane tessellation.

The main question is now to evaluate the covering probability $P(\nu, n)$. In the two-dimensional case, it is known explicitly [**?**] so the preceding proposition provides in fact the exact calculation of the distribution tail of R_M. Unfortunately, the formula for $P(\nu, n)$ when ν is a continuous measure is not easy to handle but in the particular case where ν is simply a Dirac measure at $a \in [0, 1]$ (i.e. all circular arcs have a fixed length equal to a), then it has been proved by W. L. Stevens [**?**] with very elementary arguments that for every $n \in \mathbb{N}^*$, we have

$$P(\delta_a, n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} (1 - ka)_+^{n-1} \tag{1.12}$$

where $x_+ = \max(x, 0)$ for every $x \in \mathbb{R}$. In particular, it implies the following relation for every $a \in [0, 1]$

$$\lim_{n \to \infty} \frac{1 - P(\delta_a, n)}{n(1 - a)^{n-1}} = 1.$$

Exercise 1.11. Calculate $P((1 - p)\delta_0 + p\delta_a, n)$ for $a, p \in [0, 1], n \in \mathbb{N}^*$

In higher dimensions, no closed formula is currently available for $P(\nu, n)$. The case where $\nu = \delta_a$ with $a > 1/2$ has been solved recently [**?**], otherwise bounds do exist in the particular case of a deterministic radius of the spherical caps [**?**, **?**].

In dimension two, we can use Proposition **??** in order to derive an estimation of the distribution tail which is better than (**??**).

Theorem 1.3. (Distribution tail estimate of R_M in dimension two) *For a sufficiently large r, we have*

$$2\pi r^2 e^{-\pi r^2} \leq \mathbf{P}(R_M \geq r) \leq 4\pi r^2 e^{-\pi r^2}. \tag{1.13}$$

Sketch of proof. When using (**??**), we have to estimate $P(\nu, n)$, with ν chosen as in Proposition **??**, but possibly without considering a too complicated explicit formula. In particular, since the asymptotic equivalent (**??**) for $P(\delta_a, n)$ seems to be quite simple, we aim at replacing the covering probability $P(\nu, n)$ with a covering probability $P(\delta_a, n)$ where a is equal to $1/4$, i.e. the mean of ν.

The problem is reduced to the investigation under which conditions we can compare two different covering probabilities $P(\mu_1, n)$ and $P(\mu_2, n)$ where μ_1, μ_2 are two probability measures on $[0, 1]$. We recall that μ_1 and μ_2 are said to be *ordered according to the convex order*, i.e. $\mu_1 \prec_{\text{conv}} \mu_2$, if $\langle f, \mu_1 \rangle \leq \langle f, \mu_2 \rangle$ for every convex function $f : [0, 1] \to \mathbb{R}$ [**?**] (where $\langle f, \mu_1 \rangle = \int f d\mu_1$). In particular, Jensen's inequality says that $\delta_a \prec_{\text{conv}} \nu$ and we can easily prove that $\nu \prec_{\text{conv}} \frac{1}{2}(\delta_0 + \delta_{2a})$. The next proposition shows how the convex ordering of distributions implies the ordering of the underlying covering probabilities.

Proposition 1.9. (Ordering of covering probabilities) *If $\nu_1 \prec_{\text{conv}} \nu_2$, then for every $n \in \mathbb{N}$, $P(\nu_1, n) \leq P(\nu_2, n)$.*

Exercise 1.12. Find a heuristic proof of Proposition **??**.

Thanks to this proposition and the remark above, we can write

$$P(\delta_{1/4},n) \leq P(\nu,n) \leq P((\delta_0 + \delta_{1/2})/2,n),$$

then insert the two bounds in the equality (**??**) and evaluate them with Stevens' formula (**??**). □

Remark 1.10. Numerical estimates of $\mathbf{P}(R_M \geq r)$ with the formula (**??**) indicate that $\mathbf{P}(R_M \geq r)$ should be asymptotically equivalent to the upper bound of (**??**).

1.2.1.3 Distribution conditionally on the inradius

Why should we be interested in the behaviour of the typical cell when conditioned on the value of its inradius?

First, it is one of the rare examples of conditioning of the typical cell which can be made completely explicit. Indeed, conditionally on $\{R_m \geq r\}$, any point x of the Poisson point process is at distance larger than $2r$ from o so the typical cell $C(o)$ is equal in distribution to the zero-cell associated with the bisecting hyperplanes of the segments $[o,x]$ where x is any point of a homogeneous Poisson point process in $B_{2r}(o)^c$.

Conditionally on $\{R_m = r\}$, the distribution of $C(o)$ is obtained as previously, but with an extra-bisecting hyperplane generated by a deterministic point x_0 at distance $2r$.

The second reason for investigating this particular conditioning is that a large inradius implies a large typical cell. In other words, having R_m large is a particular case of the general setting of D. G. Kendall's conjecture (see Remark **??**). But we can be more precise about how the typical cell is converging to the spherical shape. Indeed, the boundary of the polyhedron is included in an annulus between the two radii R_m and R_M and so the order of decreasing of the difference $R_M - R_m$ will provide a satisfying way of measuring the closeness of $C(o)$ to a sphere. The next theorem provides a result in this direction.

Theorem 1.4. (Asymptotic joint distribution of (R_m, R_M)) *There exists a constant* $c > 0$ *such that for every* $\frac{d-1}{d+1} < \delta < 1$, *we have*

$$\mathbf{P}(R_M \geq r + r^\delta \mid R_m = r) = \mathcal{O}(\exp\{-cr^\beta\}), \ r \to \infty, \qquad (1.14)$$

where $\beta = \frac{1}{2}[(d-1) + \delta(d+1)]$.

Sketch of proof in dimension two. The joint distribution of the couple (R_m, R_M) can be obtained explicitly via the same method as in Proposition **??**. Indeed, the quantity $\mathbf{P}(R_M \geq r + s \mid R_m = r)$ can be rewritten as the probability of not covering the unit-sphere with random i.i.d. and uniform spherical caps. The only difference lies in the common distribution of the angular diameters of the caps which will now

depend upon r since bisecting hyperplanes have to be at least at distance r from the origin. In dimension two, the covering probability can be estimated with an upper-bound due to L. Shepp [**?**], which implies the estimation (**??**).

Unfortunately, the method does not hold in higher dimensions because of the lack of information about random coverings of the sphere. Nevertheless, a different approach will be explained in the next section in order to extend (**??**) to $d \geq 3$.   □

Exercise 1.13. For $d = 2$, estimate the minimal number of sides of the Poisson-Voronoi typical cell conditioned on $\{R_m = r\}$.

Remark 1.11. This roughly means that the boundary of the cell is included in an annulus centered at the origin and of thickness of order $R_m^{-(d-1)/(d+1)}$. The next problem would be to describe the shape of the polyhedron inside this annulus. For instance, in dimension two, a regular polygon which would be exactly „inscribed" in an annulus of thickness $R_m^{-1/3}$ would have about $R_m^{2/3}$ sides. Is it the same growth rate as the number of sides of the typical cell? The next section will be devoted to this problem. In particular, we shall see that indeed, this quantity behaves roughly as if the typical cell would be a deterministic regular polygon.

1.2.2 Limit theorems for the number of hyperfaces

This section is based on arguments and results which come from a collaboration with T. Schreiber and are developed in [**?**, **?**, **?**]. When the inradius goes to infinity, the shape of the typical Poisson-Voronoi cell becomes spherical. In particular its boundary is contained in an annulus with a thickness going to zero and thus we aim at being more specific about the evolution of the geometry of the cell when R_m is large. For the sake of simplicity, we focus essentially on a particular quantity, which is the number of hyperfaces, but our methods can be generalized to investigate other characteristics, as emphasized in the final remarks.

1.2.2.1 Connection with random convex hulls in the ball

We start with the following observation: in the literature, there are more limit theorems available for random polytopes constructed as convex hulls of a Poisson point process than for typical cells of stationary tessellations (cf. Sections **??** and **??**). Models of random convex hulls have been probably considered as more natural objects to be constructed and studied. Our aim is first to connect our model of typical Poisson-Voronoi cell with a classical model of a random convex hull in the ball and then work on this possible link between the two in order to extend what is known about random polytopes and solve our current problem.

Conditionally on $\{R_m \geq r\}$, the rescaled typical cell $\frac{1}{r}C(o)$ is equal in distribution to the zero-cell of a hyperplane tessellation generated by a Poisson point process of intensity measure $(2r)^d \mathbf{1}(x \in B_1(o)^c) \, \nu_d(dx)$ [**?**]. In other words, via this scaling we

fix the inradius of the polyhedron whereas the intensity of the underlying hyperplane process outside of the inball is now the quantity which goes to infinity.

The key idea is then to apply a geometric transformation to $\frac{1}{r}C(o)$ in order to get a random convex hull inside the unit-ball $B_1(o)$. Let us indeed consider the inversion I defined by

$$I(x) = \frac{x}{\|x\|^2}, \quad x \in \mathbb{R}^d \setminus \{o\}.$$

In the following lines, we investigate the action of I on points, hyperplanes and the cell itself. The Poisson point process of intensity measure $(2r)^d \mathbf{1}(x \notin B_1(o)) \nu_d(dx)$ is sent by I to a new Poisson point process Y_r of intensity measure $(2r)^d \mathbf{1}(x \in B_1(o)) \frac{1}{\|x\|^{2d}} \nu_d(dx)$. The hyperplanes are sent to spheres containing the origin and included in the unit ball, i.e. spheres $\partial B_{\|x\|/2}(x/2)$ where x belongs to the new Poisson point process Y_r in $B_1(o)$. The boundary of the rescaled typical cell $\frac{1}{r}C(o)$ is sent to the boundary of a certain Voronoi flower, i.e. the union of balls $B_{\|x\|/2}(x/2)$ where x belongs to Y_r. In particular, the number of hyperfaces of the typical cell $C(o)$ remains unchanged after rescaling and can also be seen through the action of the inversion I as the number of portions of spheres on the boundary of the Voronoi flower in $B_1(o)$, that is the number of extreme points of the convex hull of the process Y_r. Indeed, it can be verified that the ball $B_{\|x\|/2}(x/2)$ intersects the boundary of the Voronoi flower of Y_r iff there exists a support hyperplane of the convex hull of Y_r which contains x.

1.2.2.2 Results

Let N_r be a random variable distributed as the number of hyperfaces of the typical cell $C(o)$ conditioned on $\{R_m = r\}$.

We are now ready to derive limit theorems for the behaviour of N_r when r goes to infinity:

Theorem 1.5. (Limit theorems for the number of hyperfaces) *There exists a constant $a > 0$ (known explicitly) depending only on d such that*

$$ar^{-\frac{d(d-1)}{d+1}} N_r \to 1 \quad \text{in } L^1 \text{ and a.s. as } r \to \infty.$$

Moreover, the number N_r satisfies a central limit theorem when $r \to \infty$ as well as a moderate-deviation result: for every $\varepsilon > 0$,

$$\liminf_{r \to \infty} \frac{1}{\log(r)} \log\left(-\log\left(\mathbf{P}\left\{\left|\frac{N_r}{\mathbf{E}N_r} - 1\right| \geq \varepsilon\right\}\right)\right) \geq \frac{d-1}{3d+5}.$$

Sketch of proof. The first two steps are devoted to proving the results for the number $\widetilde{N}_r$ of hyperfaces of $C(o)$ conditioned on $\{R_m \geq r\}$. In the last step, we explain how to adapt the arguments for the number N_r.

Step 1. We use the action of the inversion I to rewrite N_r as the number of vertices of the convex hull of the Poisson point process Y_r of intensity measure $(2r)^d \mathbf{1}(x \in$

$B_1(o)) \frac{1}{\|x\|^{2d}} \nu_d(dx)$ in the unit ball. Limit theorems for the number of extreme points of a homogeneous set of points in the ball are classically known in the literature: indeed, a first law of large numbers has been established in [**?**] and generalized in [**?**]. A central limit theorem has been proved in [**?**] and extended by a precise variance estimate in [**?**]. Finally, a moderate deviations-type result has been provided in [**?**] and [**?**] (see also Sections **??** and **??** for more details).

Step 2. The only problem here is that we are not in the classical setting of all these previous works since the process Y_r is not homogeneous. Nevertheless, it can be overcome by emphasizing two points: first, when $\|x\|$ is close to one, the intensity measure of Y_r is close to $(2r)^d \nu_d(dx)$ and secondly, with high probability, only the points near the boundary of the unit sphere will be useful to construct the convex hull. Indeed, for any Poisson point process of intensity measure $\lambda f(\|x\|) \nu_d(dx)$ with $f : (0,1) \to \mathbb{R}_+$ a function such that $\lim_{t \to 1} f(t) = 1$, it can be stated that the associated convex hull K_λ satisfies the following: there exist constants $c, c' > 0$ such that for every $\alpha \in (0, \frac{2}{d+1})$, we have

$$\mathbf{P}(B_{1-c\lambda^{-\alpha}}(o) \not\subseteq K_\lambda) = \mathcal{O}(\exp\{-c'\lambda^{1-\alpha(d+1)/2}\}). \qquad (1.15)$$

The asymptotic result (**??**) roughly means that all extreme points are near $\mathbb{S}^{d-1}$ and included in an annulus of thickness $\lambda^{-2/(d+1)}$. It can be shown in the following way: we consider a deterministic covering of an annulus $B_1(o) \setminus B_{1-\lambda^{-\alpha}}(o)$ with a polynomial number of full spherical caps. When the Poisson point process intersects each of these caps, its convex hull contains $B_{1-c\lambda^{-\alpha}}(o)$ where $c > 0$ is a constant. Moreover, an estimation of the probability that at least one of the caps fails to meet the point process provides the right-hand side of (**??**).

To conclude, the estimation (**??**) allows us to apply the classical limit theory of random convex hulls in the ball even if the point process Y_r is not homogeneous.

Step 3. We recall that the difference between the constructions of $C(o)$ conditioned either on $\{R_m \geq r\}$ or on $\{R_m = r\}$ is only an extra deterministic hyperplane at distance r from the origin. After the use of a rescaling and of the inversion I, we obtain that $\widetilde{N}_r$ (obtained with conditioning on $\{R_m \geq r\}$) is the number of extreme points of Y_r whereas N_r (obtained with conditioning on $\{R_m = r\}$) is the number of extreme points of $Y_r \cup \{x_0\}$ where x_0 is a deterministic point on $\mathbb{S}^{d-1}$. A supplementary extreme point on $\mathbb{S}^{d-1}$ can „erase" some of the extreme points of Y_r but it can be verified that it will not subtract more than the number of extreme points contained in a d-dimensional polyhedron. Now the growth of extreme points of random convex hulls in a polytope has been shown to be logarithmic so we can consider that the effect of the extra point x_0 is negligible (see in particular [**?**], [**?**], and [**?**] about limit theorems for random convex hulls in a fixed polytope). Consequently, results proved for $\widetilde{N}_r$ in Steps 1-2 hold for N_r as well.   □

Remark 1.12. Up to now, the bounds on the conditional distribution of the circumscribed radius (**??**) was only proved in dimension two through techniques involving covering probabilities of the circle. Now applying the action of the inversion I once again, we deduce from (**??**) the generalization of the asymptotic result (**??**) to higher dimensions.

Remark 1.13. The same type of limit theorems occurs for the Lebesgue measure of the region between the typical cell and its inball. Indeed, after application of I, this volume is equal to the μ-measure of the complementary of the Voronoi flower of the Poisson point process in the unit ball, where μ is the image of the Lebesgue measure under I. Limit theorems for this quantity have been obtained in [**?**, **?**].

In a recent paper [**?**], this work is extended in several directions, including variance estimates and a functional central limit result for the volume of the typical cell. Moreover [**?**] contains an extreme value-type convergence for R_M which adds to (**??**) by providing a three-terms expansion of $(R_M - r)$ conditionally on $\{R_m \geq r\}$, when r goes to infinity. More precisely, it is proved that there exist explicit constants $c_1, c_2, c_3 > 0$ (depending only on the dimension) such that conditionally on $\{R_m \geq r\}$, the quantity

$$\frac{2^{\frac{3d+1}{2}} \kappa_{d-1}}{d+1} r^{\frac{d-1}{2}} (R_M - r)^{\frac{d+1}{2}} - c_1 \log(r) - c_2 \log(\log(r)) - c_3$$

converges in distribution to the Gumbel law when $r \to \infty$.

References

1. Abrahamsen, P.: A review of Gaussian random fields and correlation functions. Tech. Rep. 917, Norwegian Computing Center (1997)
2. Adler, R.J., Samorodnitsky, G., Taylor, J.E.: Excursion sets of the stable random processes. Tech. Rep. 1467, ORIE (2008). URL http://hdl.handle.net/1813/9444
3. Adler, R.J., Samorodnitsky, G., Taylor, J.E.: High level excursions set geometry of non-gaussian infinitely divisible random fields (2009). URL http://arxiv.org/abs/0907.3359
4. Adler, R.J., Taylor, J.E.: Random Fields and Geometry. Springer, New York (2007)
5. Affentranger, F.: The convex hull of random points with spherically symmetric distributions. Rend. Sem. Mat. Univ. Pol. Torino **49**, 359–383 (1991)
6. Affentranger, F.: Aproximación aleatoria de cuerpos convexos. Publ. Mat. **36**, 85–109 (1992)
7. Affentranger, F., Schneider, R.: Random projections of regular simplices. Discrete Comput. Geom. **7**, 219–226 (1992)
8. Ahlswede, R., Blinovsky, V.: Lectures on Advances in Combinatorics. Springer, Berlin (2008)
9. Ahlswede, R., Daykin, D.E.: An inequality for the weights of two families of sets, their unions and intersections. Probab. Th. Rel. Fields **43**, 183–185 (1978)
10. Alam, K., Saxena, K.M.L.: Positive dependence in multivariate distributions. Comm. Statist. Theory Methods **A10**, 1183–1196 (1981)
11. Aldous, D.: The continuum random tree. II. An overview, in: Barlow, M. T., Bingham, N. H. (eds.) *Stochastic Analysis* London Math. Soc. Lecture Note Ser. **167**. Cambridge Univ. Press, Cambridge (1991)
12. Alon, N., Spencer, J.H.: The Probabilistic Method, 2nd edn. Wiley, New York (2000)
13. Alonso-Gutiérrez, D.: On the isotropy constant of random convex sets. Proc. Amer. Math. Soc. **136**, 3293–3300 (2008)
14. Anandkumar, A., Yukich, J.E., Tong, L., Swami, A.: Energy scaling laws for distributed inference in random networks. IEEE Journal on Selected Areas in Communications, Issue on Stochastic Geometry and Random Graphs for Wireless Networks **27**, 1203–1217 (2009)
15. Anderson, D.N.: A multivariate Linnik distribution. Stat. Probab. Lett. **14**, 333–336 (1992)
16. Apanasovich, T.V., Genton, M.G.: Cross-covariance functions for multivariate random fields based on latent dimensions. Biometrika **97**(1), 15–30 (2010)
17. Araujo, A., Giné, E.: The Central Limit Theorem for Real and Banach Valued Random Variables. Wiley, New York (1980)
18. Aronszajn, N.: Theory of reproducing kernels. Trans. Amer. Math. Soc. **68**, 337–404 (1950)
19. Artstein, Z., Vitale, R.A.: A strong law of large numbers for random compact sets. Ann. Probab. **3**, 879–882 (1975)
20. Askey, R.: Refinements of Abel summability for Jacobi series, in: More, C. C. (ed.) *Harmonic Analysis on Homogeneous Spaces. Proc. Symp. Pure Math. XXVI.* AMS, Providence, RI (1973)

21. Auneau, J., Jensen, E.B.V.: Expressing intrinsic volumes as rotational integrals. To appear in Adv. Appl. Probab.

22. Avram, F., Bertsimas, D.: On central limit theorems in geometrical probability. Ann. Appl. Probab. **3**, 1033–1046 (1993)

23. Baccelli, F., Blaszczyszyn, B.: Stochastic Geometry and Wireless Networks. Now Publishers, Delft (2009)

24. Baddeley, A.: Fundamentals of point process statistics, in: Baddeley, A. J., Bárány, I., Schneider, R., Weil, W. (eds.) *Stochastic Geometry - Lecture Notes in Mathematics* **1892**. Springer, Berlin (2007)

25. Baddeley, A.: Analysing spatial point patterns in R. Tech. rep., CSIRO Mathematics, Informatics and Statistics (2008). URL www.csiro.au/resources/pf16h.html

26. Baddeley, A., Bárány, I., Schneider, R., Weil, W.: Stochastic Geometry. Springer, Heidelberg (2004)

27. Baddeley, A., Turner, R.: Spatstat: an R package for analyzing spatial point patterns. Journal of Statistical Software **12**, 1–142 (2005). URL www.jstatsoft.org

28. Baddeley, A.J.: Vertical sections, in: Weil, W., Ambartzumian, R. V. (eds.) *Stochastic Geometry and Stereology (Oberwolfach 1983)*. Teubner, Berlin (1983)

29. Baddeley, A.J.: Time-invariance estimating equations. Bernoulli **6**, 783–808 (2000)

30. Baddeley, A.J., Gundersen, H.J.G., Cruz-Orive, L.M.: Estimation of surface area from vertical sections. J. Microsc. **142**, 259–276 (1986)

31. Baddeley, A.J., Jensen, E.B.V.: Stereology for Statisticians. Chapman & Hall, Boca Raton (2005)

32. Bakhtin, Y.: Limit theorems for associated random fields. Theory Probab. Appl. **54**, 678–681 (2010)

33. Bakhtin, Y.: SPDE approximation for random trees. Submitted (2010). Also available at http://arxiv.org/abs/0909.2283.

34. Bakhtin, Y.: Thermodynamic limit for large random trees. Accepted at Random Structures Algorithms (2010). Also available at http://arxiv.org/abs/0809.2974.

35. Bakhtin, Y., Bulinski, A.: Moment inequalities for sums of dependent multiindexed random variables. Fundam. Prikl. Math. **3**, 1101–1108 (1997)

36. Bakhtin, Y., Heitsch, C.: Large deviations for random trees. J. Stat. Phys. **132**(3), 551–560 (2008)

37. Bakhtin, Y., Heitsch, C.: Large deviations for random trees and the branching of RNA secondary structures. Bull. Math. Biol. **71**(1), 84–106 (2009)

38. Baltz, A., Dubhashi, D., Srivastav, A., Tansini, L., Werth, S.: Probabilistic analysis for a vehicle routing problem. random structures and algorithms. proceedings from the 12th international conference 'random structures and algorithms', august 1-5, 2005 (2007)

39. Banys, P.: CLT for linear random fields with stationary martingale–difference innovation. Lithuanian Math. J. **17**, 1–7 (2011)

40. Bárány, I.: Random polytopes in smooth convex bodies. Mathematika **39**, 81–92 (1992). Corrigendum: Mathematika **51**, 31 (2005)

41. Bárány, I.: Random polytopes, convex bodies, and approximation, in: Baddeley, A. J., Bárány, I., Schneider, R., Weil, W. (eds.) *Stochastic Geometry - Lecture Notes in Mathematics* **1892**. Springer, Berlin (2007)

42. Bárány, I.: Random points and lattice points in convex bodies. Bull. Amer. Math. Soc. **45**, 339–365 (2008)

43. Bárány, I., Buchta, C.: Random polytopes in a convex polytope, independence of shape, and concentration of vertices. Math. Ann. **297**, 467–497 (1993)

44. Bárány, I., Fodor, F., Vigh, V.: Intrinsic volumes of inscribed random polytopes in smooth convex bodies. Adv. in Appl. Probab. **42**, 605–619 (2009)

45. Bárány, I., Larman, D.G.: Convex bodies, economic cap coverings, random polytopes. Mathematika **35**, 274–291 (1988)

46. Bárány, I., Reitzner, M.: On the variance of random polytopes. Adv. Math. (2010). (to appear)

47. Bárány, I., Reitzner, M.: Poisson polytopes. Ann. Probab. **38**, 1507–1531 (2010)
48. Bárány, I., Vu, V.: Central limit theorems for Gaussian polytopes. Ann. Probab. **35**, 1593–1621 (2007)
49. Barbato, D.: FKG inequalities for Brownian motion and stochastic differential equations. Electron. Comm. Probab. **10**, 7–16 (2005)
50. Barbour, A.D., Xia, A.: Normal approximation for random sums. Adv. in Appl. Prob. **38**, 693–728 (2006)
51. Bardossy, A.: Introduction to Geostatistics. Tech. rep., University of Stuttgart, Stuttgart (1997)
52. Barndorff-Nielsen, O.E., Mikosch, T., Resnick, S.I., eds.: Lévy processes. Theory and Applications. Birkhäuser, Boston (2001)
53. Barndorff-Nielsen, O.E., Schmiegel, J.: Spatio-temporal modeling based on Lévy processes, and its applications to turbulence. Russ. Math. Surv. **59**, 65–90 (2004)
54. Baryshnikov, Y., Eichelsbacher, P., Schreiber, T., Yukich, J.E.: Moderate deviations for some point measures in geometric probability. Annales de l'Institut Henri Poincaré - Probabilités et Statistiques **44**, 442–446 (2008)
55. Baryshnikov, Y., Penrose, M., Yukich, J.E.: Gaussian limits for generalized spacings. Ann. Appl. Probab. **19**, 158–185 (2009)
56. Baryshnikov, Y., Yukich, J.E.: Gaussian limits for random measures in geometric probability. Ann. Appl. Probab. **15**, 213–253 (2005)
57. Baryshnikov, Y.M., Vitale, R.A.: Regular simplices and Gaussian samples. Discrete Comput. Geom. **11**, 141–147 (1994)
58. Baumstark, V., Last, G.: Some distributional results for Poisson-Voronoi tessellations. Adv. in Appl. Probab. **39**, 16–40 (2007)
59. Baumstark, V., Last, G.: Gamma distributions for stationary Poisson flat processes. Adv. in Appl. Probab. **41**, 911–939 (2009)
60. Beardwood, J., Halton, J.H., Hammersley, J.M.: The shortest path through many points. Proc. Camb. Philos. Soc. **55**, 229–327 (1959)
61. Berg, C., Christensen, J.P.R., Ressel, P.: Harmonic Analysis on Semigroups. Springer, Berlin (1984)
62. van den Berg, J., Kahn, J.: A correlation inequality for connection events in percolation. Ann. Probab. **29**, 123–126 (2001)
63. Besag, J.: Statistical analysis of non-lattice data. The Statistician **24**, 179–195 (1975)
64. Besag, J.: Some methods of statistical analysis for spatial data. Bull. Internat. Statist. Inst. **44**, 77–92 (1978)
65. Bickel, P., Yan, D.: Sparsity and the possibility of inference. Sankhyā **70**, 1–23 (2008)
66. Biermé, H., Meerschaert, M.M., Scheffler, H.P.: Operator scaling stable random fields. Stochastic Process. Appl. **117**, 312–332 (2007)
67. Billingsley, P.: Convergence of Probability Measures. Wiley, New York (1968)
68. Billingsley, P.: Convergence of Probability Measures, 2nd edn. Wiley Series in Probability and Statistics: Probability and Statistics. Wiley, New York (1999)
69. Bingham, N.H., Goldie, C.M.: Riesz means and self-neglecting functions. Math. Z. **199**, 443–454 (1988)
70. Bingham, N.H., Goldie, C.M., Teugels, J.L.: Regular Variation. Cambridge Univ. Press, Cambridge (1987)
71. Birkel, T.: Moment bounds for associated sequences. Ann. Probab. **16**, 1184–1193 (1988)
72. Birkhoff, G.: Lattice Theory, 3rd edn. AMS, Providence, RI (1979)
73. Bolthausen, E.: On the central limit theorem for stationary mixing random fields. Ann. Probab. **10**, 1047–1050 (1982)
74. Bonami, A., Estrade, A.: Anisotropic analysis of some Gaussian models. J. Fourier Anal. Appl. **9**, 215–236 (2003)
75. Böröczky, K.J., Fodor, F., Hug, D.: The mean width of random polytopes circumscribed around a convex body. J. Lond. Math. Soc. **81**, 499–523 (2010)
76. Böröczky, K.J., Fodor, F., Reitzner, M., Vígh, V.: Mean width of random polytopes in a reasonably smooth convex body. J. Multivariate Anal. **100** (2009)

77. Böröczky, K.J., Hoffmann, L.M., Hug, D.: Expectation of intrinsic volumes of random polytopes. Period. Math. Hungar. **57**, 143–164 (2008)
78. Böröczky, K.J., Hug, D.: Stability of the reverse Blaschke-Santaló inequality for zonoids and applications. Adv. Appl. Math. **44**, 309–328 (2010)
79. Böröczky, K.J., Schneider, R.: The mean width of circumscribed random polytopes. Canad. Math. Bull. **53**, 614–628 (2010)
80. Box, G.E.P., Jenkins, G.M., Reinsel, G.C.: Time Series Analysis, 4th edn. Wiley, Hoboken, NJ (2008)
81. Bradley, R.C.: Introduction to Strong Mixing Conditions. Vol. 1,2,3. Kendrick Press, Heber City, UT (2007)
82. Brémaud, P.: Markov Chains: Gibbs fields, Monte Carlo simulation, and queues, *Texts in Applied Mathematics*, vol. 31. Springer, New York (1999)
83. Brillinger, D.R.: Time Series. Data Analysis and Theory. Holt, Rinehart and Winston, Inc., New York (1975)
84. Brix, A., Kendall, W.S.: Simulation of cluster point processes without edge effects. Adv. in Appl. Prob. **34**, 267–280 (2002)
85. Brockwell, P.J., Davis, R.A.: Time Series: Theory and Methods, 2nd edn. Springer, New York (1991)
86. Buchta, C.: Zufällige Polyeder - Eine Übersicht, in: Hlawka, E. (ed.) *Zahlentheoretische Analysis - Lecture Notes in Mathematics*, **1114**. Springer, Berlin (1985)
87. Buchta, C.: An identity relating moments of functionals of convex hulls. Discrete Comput. Geom. **33**, 125–142 (2005)
88. Buchta, C., Reitzner, M.: The convex hull of random points in a tetrahedron: Solution of Blaschke's problem and more general results. J. Reine Angew. Math. **536**, 1–29 (2001)
89. Bulinski, A.: Inequalities for moments of sums of associated multi-indexed variables. Theory Probab. Appl. **38**, 342–349 (1993)
90. Bulinski, A.: Central limit theorem for random fields and applications, in: Skiadas, C. H. (ed.) *Advances in Data Analysis*. Birkhäuser (2010)
91. Bulinski, A.: Central limit theorem for positively associated stationary random fields. Vestnik St. Petersburg University: Mathematics **44**(2), 89–96 (2011)
92. Bulinski, A., Shashkin, A.: Rates in the central limit theorem for dependent multiindexed random vectors. J. Math. Sci. **122**, 3343–3358 (2004)
93. Bulinski, A., Shashkin, A.: Strong invariance principle for dependent random fields. In: Dynamics & Stochastics, *IMS Lecture Notes Monogr. Ser.*, vol. 48, pp. 128–143. Inst. Math. Statist., Beachwood, OH (2006)
94. Bulinski, A., Shashkin, A.: Limit Theorems for Associated Random Fields and Related Systems. World Scientific, Singapore (2007)
95. Bulinski, A., Spodarev, E., Timmermann, F.: Central limit theorems for the excursion sets volumes of weakly dependent random fields. Bernoulli p. to appear (2011)
96. Bulinski, A., Suquet, C.: Normal approximation for quasi-associated random fields. Statist. Probab. Lett. **54**, 215–226 (2001)
97. Bulinski, A.V.: Limit Theorems under Weak Dependence Conditions. MSU (1990). (in Russian)
98. Bulinski, A.V., Keane, M.S.: Invariance principle for associated random fields. J. Math. Sci. **81**, 2905–2911 (1996)
99. Bulinski, A.V., Kryzhanovskaya, N.: Convergence rate in CLT for vector-valued random fields with self-normalization. Probab. Math. Stat. **26**, 261–281 (2006)
100. Bulinski, A.V., Shabanovich, E.: Asymptotical behaviour for some functionals of positively and negatively dependent random fields. Fundam. Prikl. Math. **4**, 479–492 (1998). (in Russian)
101. Bulinski, A.V., Shiryaev, A.N.: Theory of Stochastic Processes. Fizmatlit (2005). (in Russian)
102. Bürgisser, P., Cucker, F., Lotz, M.: Coverage processes on spheres and condition numbers for linear programming. To appear in Ann. Probab. (2009)

103. Burton, R., Waymire, E.: Scaling limits for associated random measures. Ann. Probab. **13**, 1267–1278 (1985)
104. Burton, R.M., Dabrowski, A.R., Dehling, H.: An invariance principle for weakly associated random vectors. Stochastic Process. Appl. **23**, 301–306 (1986)
105. Calka, P.: Mosaïques poissoniennes de l'espace euclidian. Une extension d'un résultat de R. E. Miles. C. R. Math. Acad. Sci. Paris **332**, 557–562 (2001)
106. Calka, P.: The distributions of the smallest disks containing the Poisson-Voronoi typical cell and the Crofton cell in the plane. Adv. in Appl. Probab. **34**, 702–717 (2002)
107. Calka, P.: Tessellations, in: Kendall, W. S., Molchanov, I. (eds.) *New Perspectives in Stochastic Geometry*. Oxford Univ. Press (2010)
108. Calka, P., Schreiber, T.: Limit theorems for the typical Poisson-Voronoi cell and the Crofton cell with a large inradius. Ann. Probab. **33**, 1625–1642 (2005)
109. Calka, P., Schreiber, T.: Large deviation probabilities for the number of vertices of random polytopes in the ball. Adv. in Appl. Probab. **38**, 47–58 (2006)
110. Calka, P., Schreiber, T., Yukich, J.E.: Brownian limits, local limits, extreme value and variance asymptotics for convex hulls in the unit ball (2012). (to appear)
111. Carlsson, G.: Topology and data. Bull. Amer. Math. Soc. (N.S.) **46**, 255–308 (2009)
112. Chambers, J.M., Mallows, C., Stuck, B.W.: A method for simulating stable random variables. J. Amer. Statist. Assoc. **71**, 340–344 (1976)
113. Chan, G., Wood, A.T.A.: An algorithm for simulating stationary Gaussian random fields. J. R. Statist. Soc. Ser. C **46**, 171–181 (1997)
114. Chatterjee, S.: A new method of normal approximation. Ann. Probab. **36**, 1584–1610 (2008)
115. Chayes, L., Lei, H.K.: Random cluster models on the triangular lattice. J. Stat. Phys. **122**, 647–670 (2006)
116. Chazal, F., Guibas, L., Oudot, S., Skraba, P.: Analysis of scalar fields over point cloud data. Preprint (2007)
117. Chazal, F., Oudot, S.: Towards persistence-based reconstruction in euclidean spaces. ACM Symposium on Computational Geometry **232** (2008)
118. Chen, L., Shao, Q.M.: Normal approximation under local dependence. Ann. Probab. **32**, 1985–2028 (2004)
119. Chen, T., Huang, T.S.: Region based hidden Markov random field model for brain MR image segmentation. World Academy of Sciences. Engineering and Technology **4**, 233–236 (2005)
120. Chilès, J.P., Delfiner, P.: Geostatistics: Modeling Spatial Uncertainty. Wiley, New York (1999)
121. Chow, Y.S.: Delayed sums and Borel summability of independent, identically distributed random variables. Bull. Inst. Math. Acad. Sinica **1**, 207–220 (1973)
122. Chow, Y.S., Lai, T.L.: Some one-sided theorems on the tail distribution of sample sums with applications to the last time and largest excess of boundary crossings. Trans. Amer. Math. Soc. **208**, 51–72 (1975)
123. Chow, Y.S., Teicher, H.: Probability Theory: Independence, Interchangeability, Martingales. Springer Texts in Statistics. Springer, New York (2003)
124. Christofidis, T.C., Vaggelatou, E.: A connection between supermodular ordering and positive/negative association. J. Multivariate Anal. **88**, 138–151 (2004)
125. Cohen, S., Lacaux, C., Ledoux, M.: A general framework for simulation of fractional fields. Stoch. Proc. Appl. **118**, 1489–1517 (2008)
126. Cont, R., Tankov, P.: Financial Modeling with Jump Processes. Chapman & Hall, Boca Raton (2004)
127. Costa, J., Hero III, A.: Geodesic entropic graphs for dimension and entropy estimation in manifold learning. IEEE Trans. Signal Process. **58**, 2210–2221 (2004)
128. Costa, J., Hero III, A.: Determining intrinsic dimension and entropy of high-dimensional shape spaces, in: Krim, H. and Yezzi, A. (eds.) *Statistics and Analysis of Shapes*. Birkhäuser (2006)
129. Cowan, R.: The use of the ergodic theorems in random geometry. Adv. in Appl. Prob. **10**(suppl.), 47–57 (1978)

130. Cowan, R.: Properties of ergodic random mosaic processes. Math. Nachr. **97**, 89–102 (1980)
131. Cox, J.T., Grimmett, G.: Central limit theorems for associated random variables and the percolation model. Ann. Probab. **12**, 514–528 (1984)
132. Cressie, N.A.C.: Statistics for Spatial Data, 2nd edn. Wiley, New York (1993)
133. Cruz-Orive, L.M.: A new stereological principle for test lines in threedimensional space. J. Microsc. **219**, 18–28 (2005)
134. Csörgő, M., Révész, P.: Strong Approximations in Probability and Statistics. Academic Press, New York (1981)
135. Dafnis, N., Giannopoulos, A., Guédon, O.: On the isotropic constant of random polytopes. Adv. Geom. **10**, 311–322 (2010)
136. Daley, D.J., Vere-Jones, D.: An Introduction to the Theory of Point Processes. Springer (1988)
137. Daley, D.J., Vere-Jones, D.: An introduction to the theory of point processes. Vol. I and II. Probability and its Applications (New York). Springer, New York (2003/2008)
138. Davy, P.J.: Stereology: A Statistical Viewpoint. Ph.D. thesis, Austral. National Univ. (1978)
139. Davy, P.J., Miles, R.E.: Sampling theory for opaque spatial specimens. J. R. Stat. Soc. Ser. B Stat. Methodol. **39**, 56–65 (1977)
140. Davydov, Y., Molchanov, I., Zuyev, S.: Stable distributions and harmonic analysis on convex cones. C. R. Math. Acad. Sci. Paris **344**, 321–326 (2007)
141. Davydov, Y., Molchanov, I., Zuyev, S.: Strictly stable distributions on convex cones. Electron. J. Probab. **13**, 259–321 (2008)
142. Davydov, Y., Molchanov, I., Zuyev, S.: Stability for random measures, point processes and discrete semigroups. to appear in:. Bernoulli **16** (2010)
143. Davydov, Y.A.: Convergence of distributions generated by stationary stochastic processes. Theory Probab. Appl. **13**, 691–696 (1968)
144. Dedecker, J.: A central limit theorem for stationary random fields. Probab. Theor. Relat. Fields **110**, 397–437 (1998)
145. Dedecker, J.: Exponential inequalities and functional central limit theorems for a random field. ESAIM Probab. Statist. **5**, 77–104 (2001)
146. Dedecker, J., Doukhan, P., Lang, G., León, R., José, R., Louhichi, S., Prieur, C.: Weak dependence: With Examples and Applications. Springer, Berlin (2008)
147. Dedecker, J., Louhichi, S.: Convergence to infinitely divisible distributions with finite variance for some weakly dependent sequences. ESAIM: Prob. Stat. **9**, 38–73 (2005)
148. Dietrich, C.R., Newsam, G.N.: A fast and exact method for multidimensional Gaussian stochastic simulations. Water Resour. Res. **29**, 2861–2869 (1993)
149. Dietrich, C.R., Newsam, G.N.: Fast and exact simulation of stationary Gaussian processes through circulant embedding of the covariance matrix. J. Sci. Comput. **18**, 1088–1107 (1997)
150. Diggle, P.J.: A point process modelling approach to raised incidence of a rare phenomenon in the vicinity of a prespecified point. J. Roy. Statist. Soc. Ser. A **153**, 349–362 (1990)
151. Diggle, P.J.: Statistical Analysis of Spatial Point Patterns, 2nd edn. Hodder Arnold, London (2003)
152. Dimitrakopoulos, R., Mustapha, H., Gloaguen, E.: High-order statistics of spatial random fields: Exploring spatial cumulants for modeling complex non-gaussian and non-linear phenomena. Math. Geosci. **42**, 65–99 (2010)
153. Donoho, D., Grimes, C.: Hessian eigenmaps: locally linear embedding techniques for high dimensional data. Proc. Nat. Acad. of Sci. **100**, 5591–5596 (2003)
154. Donoho, D.L., Tanner, J.: Counting faces of randomly projected polytopes when the projection radically lowers dimension. J. Amer. Math. Soc. **22**, 1–53 (2009)
155. Donoho, D.L., Tanner, J.: Exponential bounds implying construction of compressed sensing matrices, error-correcting codes, and neighborly polytopes by random sampling. IEEE Trans. Inform. Theory **56**, 2002–2016 (2010)
156. Doukhan, P.: Mixing, *Lecture Notes in Statistics*, vol. 85. Springer, New York (1994)
157. Doukhan, P., Louhichi, S.: A new weak dependence condition and applications to moment inequalities. Stochastic Process. Appl. **84**, 313–342 (1999)

158. Doukhan, P., Oppenheim, G., Taqqu, M.e.: Theory and Applications of Long-range Dependence. Birkhäuser, Boston (2003)

159. Drinkwater, M.J., Parker, Q.A., Proust, D., Slezak, E., Quintana, H.: The large scale distribution of galaxies in the Shapley supercluster. Publications of the Astronomical Society of Australia **21**, 89–96 (2004). DOI 10.1071/AS03057

160. Dryden, I.L., Farnoosh, R., Taylor, C.C.: Image segmentation using Voronoi polygons and MCMC, with application to muscle fibre images. J. Appl. Stat. **33**, 609–622 (2006)

161. Dvoretzky, A., Robbins, H.: On the "parking" problem. MTA Mat. Kut. Int. Köl. (Publications of the Math. Res. Inst. of the Hungarian Academy of Sciences) **9**, 209–225 (1964)

162. Dynkin, E.B.: Markov processes. Vols. I, II, *Translated with the authorization and assistance of the author by J. Fabius, V. Greenberg, A. Maitra, G. Majone. Die Grundlehren der Mathematischen Wissenschaften, Bände 121*, vol. 122. Academic Press, New York (1965)

163. Efron, B., Stein, C.: The jackknife estimate of variance. Ann. Statist. **9**, 586–596 (1981)

164. Ellis, R.S.: Entropy, large deviations, and statistical mechanics. Classics in Mathematics. Springer, Berlin (2006). Reprint of the 1985 original

165. Erdős, P., Rényi, A.: On a new law of large numbers. J. Analyse Math. **23**, 103–111 (1970)

166. Esary, J.D., Proschan, F., Walkup, D.W.: Association of random variables, with applications. Ann. Math. Statist. **38**, 1466–1474 (1967)

167. Ethier, S.N., Kurtz, T.G.: Markov processes. Characterization and convergence. Wiley Series in Probability and Mathematical Statistics: Probability and Mathematical Statistics. Wiley, New York (1986)

168. Evans, S.N.: Two representations of a conditioned superprocess. Proc. Roy. Soc. Edinburgh Sect. A **123**(5), 959–971 (1993)

169. Evans, S.N., Perkins, E.: Measure-valued Markov branching processes conditioned on nonextinction. Israel J. Math. **71**(3), 329–337 (1990)

170. Federer, H.: Curvature measures. Trans. Amer. Math. Soc. **93**, 418–491 (1959)

171. Feller, W.: An extension of the law of the iterated logarithm to variables without variance. J. Math. Mech. **18**, 343–355 (1968)

172. Feller, W.: An Introduction to Probability Theory and its Applications. Wiley, New York (1971)

173. Fleischer, F., Gloaguen, C., Schmidt, H., Schmidt, V., Schweiggert, F.: Simulation algorithm of typical modulated Poisson–Voronoi cells and application to telecommunication network modelling. Japan J. Indust. Appl. Math. **25**, 305–330 (2008)

174. Fleischer, F., Gloaguen, C., Schmidt, V., Voss, F.: Simulation of the typical Poisson-Voronoi-Cox-Voronoi cell. J. Stat. Comput. Simul. **79**, 939–957 (2009)

175. Fortuin, C., Kasteleyn, P., Ginibre, J.: Correlation inequalities on some partially ordered sets. Comm. Math. Phys. **22**, 89–103 (1971)

176. Foss, S., Zuyev, S.: On a Voronoi aggregative process related to a bivariate Poisson process. Adv. in Appl. Probab. **28**, 965–981 (1996)

177. Gaetan, C., Guyon, X.: Spatial Statistics and Modeling. Springer, Berlin (2010)

178. Gatzouras, D., Giannopoulos, A.: A large deviations approach to the geometry of random polytopes. Mathematika **53**, 173–210 (2006)

179. Gatzouras, D., Giannopoulos, A., Markoulakis, N.: Lower bound for the maximal number of facets of a 0/1 polytope. Discrete Comput. Geom. **34**, 331–349 (2005)

180. Gelfand, A., Fuentes, M., Diggle, P.J.: Handbook of Spatial Statistics. CRC Press, London (2010). In press

181. Genton, M.G.: Highly robust variogram estimation. Math. Geol. **30**, 213–221 (1998)

182. Genton, M.G.: Variogram fitting by generalized least squares using an explicit formula for the covariance structure. Math. Geol. **30**, 323–345 (1998)

183. Genton, M.G.: Robustness problems in the analysis of spatial data, in: Moore, M. (ed.) *Spatial Statistics: Methodological Aspects and Applications*. Springer, New York (2001)

184. Georgii, H.O.: Gibbs Measures and Phase Transitions. Walter de Gruyter, Berlin (1988)

185. Gerstein, M., Tsai, J., Levitt, M.: The volume of atoms on the protein surface: calculated from simulation, using Voronoi polyhedra. J. Mol. Biol. **249**, 955–966 (1995)

186. Gilbert, E.N.: Random subdivisions of space into crystals. Ann. Math. Statist. **33**, 958–972 (1962)
187. Gilbert, E.N.: The probability of covering a sphere with n circular caps. Biometrika **52**, 323–330 (1965)
188. Gloaguen, C., Fleischer, F., Schmidt, H., Schmidt, V.: Simulation of typical Cox-Voronoi cells with a special regard to implementation tests. Math. Methods Oper. Res. **62**, 357–373 (2005)
189. Gloaguen, C., Schmidt, V., Voss, F.: Parametric distance distributions for fixed access network analysis and planning. In: Proceedings of the 21st International Teletraffic Congress, pp. 357–373. Paris (2009)
190. Gneiting, T.: Radial positive definite functions generated by Euclid's hat. J. Multivariate Anal. **69**, 88–119 (1999)
191. Gneiting, T.: Nonseparable, stationary covariance functions for space-time data. J. Am. Statist. Assoc. **97**, 590–600 (2002)
192. Gneiting, T., Genton, M.G., Guttorp, P.: Geostatistical space-time models, stationarity, separability and full symmetry. In: B. Finkenstaedt, L. Held, V. Isham (eds.) Statistics of Spatio-Temporal Systems, Monographs in Statistics and Applied Probability, pp. 151–75. Chapman and Hall/CRC Press (2007)
193. Gneiting, T., Sasvári, Z.: The characterization problem for isotropic covariance functions. Math. Geol. **31**, 105–111 (1999)
194. Gneiting, T., Ševčíková, H., Percival, D.B., Schlather, M., Jiang, Y.: Fast and exact simulation of large Gaussian lattice systems in R^2: Exploring the limits. J. Comput. Graph. Statist. **15**, 483–501 (2006)
195. Goldman, A.: Le spectre de certaines mosaïques poissoniennes du plan et l'enveloppe convexe du pont brownien. Probab. Theory Related Fields **105**, 57–83 (1996)
196. Goldman, A.: Sur une conjecture de D. G. Kendall concernant la cellule de Crofton du plan et sur sa contrepartie brownienne. Ann. Probab. **26**, 1727–1750 (1998)
197. Goldman, A., Calka, P.: On the spectral function of the Poisson-Voronoi cells. Ann. Inst. H. Poincaré Probab. Statist. **39**, 1057–1082 (2003)
198. Götze, F., Hipp, C.: Asymptotic expansions for potential functions of i.i.d. random fields. Probab. Theory Related Fields **82**, 349–370 (1989)
199. Goudsmit, S.: Random distribution of lines in a plane. Rev. Modern Phys. **17**, 321–322 (1945)
200. Grimmett, G.: A theorem about random fields. Bull. London Math. Soc. **5**, 81–84 (1973)
201. Grimmett, G.: Probability Theory and Phase Transition. Kluwer, Dordrecht (1994)
202. Grimmett, G.: Percolation, vol. 321, 2nd edn. Springer, Berlin (1999)
203. Groemer, H.: On the mean value of the volume of a random polytope in a convex set. Arch. Math. **25**, 86–90 (1974)
204. Groemer, H.: On the extension of additive functionals on classes of convex sets. Pacific J. Math. **75**, 397–410 (1978)
205. Gruber, P.: Convex and Discrete Geometry. Springer, Berlin (2007)
206. Gruber, P.M.: Comparisons of best and random approximations of convex bodies by polytopes. Rend. Circ. Mat. Palermo (2) Suppl. **50**, 189–216 (1997)
207. Gual-Arnau, X., Cruz-Orive, L.M.: A new expression for the density of totally geodesic submanifolds in space forms, with stereological applications. Differ. Geom. Appl. **27**, 124–128 (2009)
208. Guo, H., Lim, C.Y., Meerschaert, M.M.: Local Whittle estimator for anisotropic random fields. J. Multivar. Anal. **100**, 993–1028 (2009)
209. Gut, A.: Probability: A Graduate Course, corr. 2nd printing. Springer, New York (2007)
210. Gut, A., Jonsson, F., Stadtmüller, U.: Between the LIL and LSL. Bernoulli **16**, 1–22 (2010)
211. Gut, A., Stadtmüller, U.: Laws of the single logarithm for delayed sums of random fields. Bernoulli **14**, 249–276 (2008)
212. Gut, A., Stadtmüller, U.: Laws of the single logarithm for delayed sums of random fields. II. J. Math. Anal. Appl. **346**, 403–414 (2008)

213. Gut, A., Stadtmüller, U.: An asymmetric Marcinkiewcz-Zygmund LLN for random fields. Statist. Probab. Letters **79**, 1016–1020 (2009)
214. Gut, A., Stadtmüller, U.: On the LSL for random fields. J. Theoret. Probab. (2010). (to appear)
215. Guyon, X.: Random Fields on a Network: Modeling, Statistics, and Applications. Springer, New York (1995)
216. de Haan, L., Ferreira, A.: Extreme Value Theory: an Introduction. Springer, New York (2006)
217. de Haan, L., Pereira, T.: Spatial extremes: models for the stationary case. Ann. Statist. **34**(1), 146–168 (2006)
218. de Haan, L., Zhou, C.: On extreme value analysis of a spatial process. REVSTAT **6**(1), 71–81 (2008)
219. Hadwiger, H.: Vorlesungen über Inhalt, Oberfläche und Isoperimetrie. Springer, Berlin (1957)
220. Haenggi, M., Andrews, J.G., Baccelli, F., Dousse, O., Franceschetti, M.: Stochastic geometry and random graphs for the analysis and design of wireless networks. IEEE Journal on Selected Areas in Communications **27**, 1029–1046 (2009)
221. Hall, P.: On the coverage of k-dimensional space by k-dimensional spheres. Ann. Probab. **13**, 991–1002 (1985)
222. Harris, T.E.: A lower bound for the critical probability in a certain percolation process. Proc. Cambridge Philos. Soc. **56**, 13–20 (1960)
223. Hartman, P., Wintner, A.: On the law of the iterated logarithm. Amer. J. Math. **63**, 169–176 (1941)
224. Hartzoulaki, M., Paouris, G.: Quermassintegrals of a random polytope in a convex body. Arch. Math. **80**, 430–438 (2003)
225. Heesch, D., Petrou, M.: Non-Gibbsian Markov random fields models for contextual labelling of structured scenes. In: Proc. British Machine Vision Conf., pp. 930–939 (2007)
226. Heinrich, L.: Asymptotic expansions in the central limit theorem for a special class of m-dependent random fields. I. Math. Nachr. **134**, 83–106 (1987)
227. Heinrich, L.: Asymptotic Gaussianity of some estimators for reduced factorial moment measures and product densities of stationary Poisson cluster processes. Statistics **19**, 87–106 (1988)
228. Heinrich, L.: Asymptotic expansions in the central limit theorem for a special class of m-dependent random fields. II. Lattice case. Math. Nachr. **145**, 309–327 (1990)
229. Heinrich, L.: Goodness-of-fit tests for the second moment function of a stationary multidimensional Poisson process. Statistics **22**, 245–268 (1991)
230. Heinrich, L.: On existence and mixing properties of germ-grain models. Statistics **23**, 271–286 (1992)
231. Heinrich, L.: Normal approximation for some mean-value estimates of absolutely regular tessellations. Math. Methods Statist. **3**, 1–24 (1994)
232. Heinrich, L.: Gaussian limits of multiparameter empirical k-functions of spatial poisson processes (in preparation) (2010)
233. Heinrich, L., Liebscher, E.: Strong convergence of kernel estimators for product densities of absolutely regular point processes. J. Nonparametr. Statist. **8**, 65–96 (1997)
234. Heinrich, L., Molchanov, I.S.: Central limit theorem for a class of random measures associated with germ-grain models. Adv. in Appl. Probab. **31**, 283–314 (1999)
235. Heinrich, L., Moser, M.: Limit theorems for empirical mark covariance functions of stationary marked point processes (in preparation) (2010)
236. Heinrich, L., Pawlas, Z.: Weak and strong convergence of empirical distribution functions from germ-grain processes. Statistics **42**, 49–65 (2008)
237. Heinrich, L., Prokešová, M.: On estimating the asymptotic variance of stationary point processes. Methodol. Comput. Appl. Probab. **12**, 451–471 (2010)
238. Heinrich, L., Schmidt, H., Schmidt, V.: Central limit theorems for Poisson hyperplane tessellations. Ann. Appl. Probab. **16**, 919–950 (2006)
239. Hellmund, G., Prokešová, M., Jensen, E.V.: Lévy-based Cox point processes. Adv. in Appl. Prob. **40**, 603–629 (2008)

240. Herglotz, G.: Über Potenzreihen mit positivem, reellen Teil im Einheitskreis. Ber. Verh. Sächs. Akad. Wiss. **63**, 501–511 (1911)
241. Hero, A.O., Ma, B., Michel, O., Gorman, J.: Applications of entropic spanning graphs. IEEE Signal Processing Magazine **19**, 85–95 (2002)
242. Herrndorf, N.: An example on the central limit theorem for associated sequences. Ann. Probab. **12**, 912–917 (1984)
243. Heyde, C.C., Gay, R.: Smoothed periodogram asymptotics and estimation for processes and fields with possible long–range dependence. Stochastic Process. Appl. **45**, 169–182 (1993)
244. Holley, R.: Remarks on the FKG inequalities. Comm. Math. Phys. **36**, 227–231 (1974)
245. Hsing, T.: On the asymptotic distribution of the area outside a random convex hull in a disk. Ann. Appl. Probab. **4**, 478–493 (1994)
246. Hueter, I.: Limit theorems for the convex hull of random points in higher dimensions. Trans. Amer. Math. Soc. **351**, 4337–4363 (1999)
247. Hug, D.: Contributions to affine surface area. Manuscripta Math. **91**, 283–301 (1996)
248. Hug, D.: Curvature relations and affine surface area for a general convex body and its polar. Results Math. **29**, 233–248 (1996)
249. Hug, D.: Random mosaics, in: in: Baddeley, A. J., Bárány, I., Schneider, R., Weil, W. (eds.) *Stochastic Geometry - Lecture Notes in Mathematics* **1892**. Springer, Berlin (2007)
250. Hug, D., Munsonius, O., Reitzner, M.: Asymptotic mean values of Gaussian polytopes. Contr. Algebra Geom. **45**, 531–548 (2004)
251. Hug, D., Reitzner, M.: Gaussian polytopes: variances and limit theorems. Adv. in Appl. Probab. **37**, 297–320 (2005)
252. Hug, D., Reitzner, M., Schneider, R.: Large Poisson–Voronoi cells and Crofton cells. Adv. in Appl. Probab. **36**, 667–690 (2004)
253. Hug, D., Reitzner, M., Schneider, R.: The limit shape of the zero cell in a stationary Poisson hyperplane tessellation. Ann. Probab. **32**, 1140–1167 (2004)
254. Hug, D., Schneider, R.: Large cells in Poisson-Delaunay tessellations. Discrete Comput. Geom. **31**, 503–514 (2004)
255. Hug, D., Schneider, R.: Large typical cells in Poisson-Delaunay mosaics. Rev. Roumaine Math. Pures Appl. **50**, 657–670 (2005)
256. Hug, D., Schneider, R.: Asymptotic shapes of large cells in random tessellations. Geom. Funct. Anal. **17**, 156–191 (2007)
257. Hug, D., Schneider, R.: Typical cells in Poisson hyperplane tessellations. Discrete Comput. Geom. **38**, 305–319 (2007)
258. Hug, D., Schneider, R.: Large faces in Poisson hyperplane mosaics. Ann. Probab. **38**, 1320–1344 (2010)
259. Hug, D., Schneider, R.: Faces of Poisson-Voronoi mosaics. Probab. Theory Rel. Fields. **151**, 125–151 (2011)
260. Hug, D., Schneider, R.: Faces with given directions in anisotropic Poisson hyperplane tessellations. Adv. in Appl. Probab. **43**, 308–321 (2011)
261. Ibragimov, I.A., Linnik, Y.V.: Independent and stationary sequences of random variables. Wolters-Noordhoff, Groningen (1971)
262. Illian, J., Penttinen, A., Stoyan, D., Stoyan, H.: Statistical Analysis and Modelling of Spatial Point Patterns. Wiley, Chichester (2008)
263. Itô, K., McKean Jr., H.P.: Diffusion Processes and their Sample Paths. Springer, Berlin (1974). 2nd printing, corrected, Die Grundlehren der mathematischen Wissenschaften, Band 125
264. Ivanoff, G.: Central limit theorems for point processes. Stochastic Process. Appl. **12**, 171–186 (1982)
265. Ivanov, A.V., Leonenko, N.N.: Statistical Analysis of Random Fields. Kluwer, Dordrecht (1989)
266. Jakubowski, A.: Minimal conditions in p–stable limit theorems. Stochastic Process. Appl. **44**, 291–327 (1993)
267. Jakubowski, A.: Minimal conditions in p–stable limit theorems II. Stochastic Process. Appl. **68**, 1–20 (1997)

268. Janzer, H.S., Raudies, F., Neumann, H., Steiner, F.: Image processing and feature extraction from a perspective of computer vision and physical cosmology, in: Arendt, W., Schleich, W. (eds.) *Mathematical Analysis of Evolution, Information and Complexity*. Wiley (2009)

269. Jenish, N., Prucha, I.R.: Central limit theorems and uniform laws of large numbers for arrays of random fields. J. Econometrics **150**, 86–98 (2009)

270. Joag-Dev, K., Proschan, F.: Negative association of random variables, with applications. Ann. Statist. **11**, 286–295 (1983)

271. Jolivet, E.: Central limit theorem and convergence of empirical processes for stationary point processes, in: Bartfai, P., Tomko, J. (eds.) *Point Processes and Queueing Problems*. North-Holland, Amsterdam (1981)

272. Jolivet, E.: Upper bound of the speed of convergence of moment density estimators for stationary point processes. Metrika **31**, 349–360 (1984)

273. Journel, A.G., Huijbregts, C.J.: Mining Geostatistics. Academic Press, London (1978)

274. Judge, G.G., Griffiths, W.E., Hill, R.C., Lee, T.C.: The Theory and Practice of Econometrics. Wiley, New York (1980)

275. Kabluchko, Z., Schlather, M., de Haan, L.: Stationary max-stable fields associated to negative definite functions. Ann. Probab. **37**, 2042–2065 (2009)

276. Kac, M.: Can one hear the shape of a drum? Amer. Math. Monthly **73**, 1–23 (1966)

277. Kailath, T.: A theorem of I. Schur and its impact on modern signal processing, in: Gohberg, I. (ed.) *Schur Methods in Operator Theory and Signal Processing, Vol. I*. Birkhäuser, Basel (1986)

278. Kallenberg, O.: Random Measures, 4th edn. Academic Press, New York (1986)

279. Kallenberg, O.: Foundations of Modern Probability, 2nd edn. Springer, New York (2002)

280. Kaltenbach, F.J.: Asymptotisches Verhalten zufälliger konvexer Polyeder. Doctoral thesis, Albert-Ludwigs-Universität, Freiburg i. Br. (1990)

281. Karatzas, I., Shreve, S.E.: Methods of mathematical finance, *Applications of Mathematics (New York)*, vol. 39. Springer, New York (1998)

282. Karcher, W.: A central limit theorem for the volume of the excursion sets of associated stochastically continuous stationary random fields. ESAIM Probab. Stat. (2012). (to appear)

283. Karcher, W., Scheffler, H.P., Spodarev, E.: Efficient simulation of stable random fields and its applications. In: V. Capasso, G. Aletti, A. Micheletti (eds.) Stereology and Image Analysis. Ecs10: Proceedings of The 10th European Congress of ISS, vol. 4, pp. 63–72. ESCULAPIO Pub. Co., Bologna, Italy (2009)

284. Karcher, W., Scheffler, H.P., Spodarev, E.: Simulation of infinitely divisible random fields. (Preprint) (2009)

285. Karr, A.F.: Point Processes and their Statistical Inference. Dekker, New York (1986)

286. Kendall, D.G.: Foundations of a theory of random sets, in: Harding, E. F., Kendall, D. G. (eds.), *Stochastic Geometry*. Wiley, New York (1974)

287. Kent, J.T., Wood, A.T.A.: Estimating fractal dimension of a locally self-similar Gaussian process by using increments. J. R. Statist. Soc. B **59**, 679–699 (1997)

288. Kesten, H.: Subdiffusive behavior of random walk on a random cluster. Ann. Inst. H. Poincaré Probab. Statist. **22**(4), 425–487 (1986)

289. Kesten, H., Lee, S.: The central limit theorem for weighted minimal spanning trees on random points. Ann. Appl. Probab. **6**, 495–527 (1996)

290. Khoshnevisian, D.: Multiparameter Processes. An Introduction to Random Fields. Springer, New York (2002)

291. Kinderman, R., Snell, J.L.: Markov Random Fields and their Applications, *Contemporary Math.*, vol. 1. AMS, Providence, RI (1980)

292. Kingman, J.F.C.: Poisson Processes, Oxford Studies in Probability. Oxford Univ. Press (1993)

293. Kirby, M.: Geometric Data Analysis: An Empirical Approach to Dimensionality Reduction and the Study of Patterns. Wiley (2001)

294. Kitanidis, P.K.: Introduction to Geostatistics: Applications to Hydrogeology. Cambridge Univ. Press, New York (1997)

295. Klain, D.A.: A short proof of hadwiger's characterization theorem. Mathematika **42**, 329–339 (1995)
296. Klain, D.A., Rota, G.C.: Introduction to Geometric Probability. Cambridge Univ. Press, Cambridge (1997)
297. Klartag, B.: An isomorphic version of the slicing problem. J. Funct. Anal. **218**, 372–394 (2005)
298. Klartag, B.: On convex perturbations with a bounded isotropic constant. Geom. Funct. Anal. **16**, 1274–1290 (2006)
299. Klartag, B., Kozma, G.: On the hyperplane conjecture for random convex sets. Israel J. Math. **170**, 253–268 (2009)
300. Klesov, O.I.: The Hájek-Rényi inequality for random fields and the strong law of large numbers. Theory Probab. Math. Statist. **22**, 63–71 (1981)
301. Klesov, O.I.: Strong law of large numbers for multiple sums of independent, identically distributed random variables. Math. Notes **38**, 1006–1014 (1985)
302. Kolmogorov, A.N.: Wienersche Spiralen und einige andere interessante Kurven in Hilbertschem Raum. (Dokl.) Acad. Sci. USSR **26**, 115–118 (1940)
303. Koo, Y., Lee, S.: Rates of convergence of means of Euclidean functionals. J. Theoret. Probab. **20**, 821B–841 (2007)
304. Koralov, L.B., Sinai, Y.G.: Theory of Probability and Random Processes. Springer, New York (2009)
305. Kotz, S., Kozubowski, T.J., Podgorski, K.: The Laplace Distribution and Generalizations. Birkhäuser, Boston (2001)
306. Kovalenko, I.N.: A simplified proof of a conjecture of D. G. Kendall concerning shapes of random polygons. J. Appl. Math. Stochastic Anal. **12**, 301–310 (1999)
307. Kratz, M., Leon, J.: Central limit theorems for level functionals of stationary Gaussian processes and fields. J. Theor. Probab. **14**, 639–672 (2001)
308. Kratz, M., Leon, J.: Level curves crossings and applications for Gaussian models. Extremes **13**, 315–351 (2010)
309. Kryzhanovskaya, N.Y.: A moment inequality for sums of multi-indexed dependent random variables. Math. Notes **83**, 770–782 (2008)
310. Kunita, H.: Stochastic flows and stochastic differential equations, *Cambridge Studies in Advanced Mathematics*, vol. 24. Cambridge Univ. Press, Cambridge (1997). Reprint of the 1990 original
311. Kwapień, S., Woyczyński, W.A.: Random Series and Stochastic Integrals: Single and Multiple. Birkhäuser, Boston (1992)
312. Lai, T.L.: Limit theorems for delayed sums. Ann. Probab. **2**, 432–440 (1974)
313. Lantuejoul, C.: Geostatistical Simulation – Models and Algorithms. Springer, Berlin (2002)
314. Lanzinger, H., Stadtmüller, U.: Maxima of increments of partial sums for certain subexponential distributions. Stochastic Process. Appl. **86**, 307–322 (2000)
315. Lao, W., Mayer, M.: *U*-max-statistics. J. Multivariate Anal. **99**, 2039–2052 (2008)
316. Last, G.: Stationary partitions and Palm probabilities. Adv. in Appl. Probab. **38**, 602–620 (2006)
317. Lautensack, C., Zuyev, S.: Random Laguerre tessellations. Adv. in Appl. Probab. **40**, 630–650 (2008)
318. Lawson, A.B., Denison, D.G.T.: Spatial Cluster Modelling: An Overview. In: Spatial Cluster Modelling, pp. 1–19. Chapman & Hall (2002)
319. Lee, M.L.T., Rachev, S.T., Samorodnitsky, G.: Association of stable random variables. Ann. Probab. **18**, 1759–1764 (1990)
320. Lehmann, E.L.: Some concepts of dependence. Ann. Math. Statist. **37**, 1137–1153 (1966)
321. Leonenko, N., Pronzato, L., Savani, V.: A class of rényi information estimators for multidimensional densities. Ann. Statist. **36**, 2153–2182 (2008)
322. Levina, E., Bickel, P.J.: Maximum likelihood estimation of intrinsic dimension, in: Saul, L. K., Weiss, Y., Bottou, L. (eds.) *Advances in NIPS* **17** (2005)
323. Lewis, T.: Limit theorems for partial sums of quasi-associated random variables, in: Szysskowicz, B. (ed.) *Asymptotic methods in Probability and Statistics*. Elsevier (1998)

324. Li, S.Z.: Markov Random Fields Modeling in Computer Vision. Springer, New York (2001)
325. Lifshits, M.A.: Partitioning of multidimensional sets. In: Rings and Modules. Limit Theorems of Probability Theory, No. 1, pp. 175–178. Leningrad. Univ., Leningrad (1985). (in Russian)
326. Liggett, T.M.: Interacting Particle Systems. Classics in Mathematics. Springer, New York (2005)
327. Liggett, T.M.: Conditional association and spin systems. ALEA Lat. Am. J. Probab. Math. Stat. **1**, 1–19 (2006)
328. Lim, S.C., Teo, L.P.: Gaussian fields and Gaussian sheets with generalized Cauchy covariance structure. Stoch. Proc. Appl. **119**, 1325–1356 (2009)
329. Lindquist, B.H.: Association of probability measures on partially ordered spaces. J. Multivariate Anal. **26**, 11–132 (1988)
330. Litvak, A.E., Pajor, A., Rudelson, M., Tomczak-Jaegermann, N.: Smallest singular value of random matrices and geometry of random polytopes. Adv. Math. **195**, 491–523 (2005)
331. Loève, M.: Probability Theory. Van Nostrand Co., Inc., Princeton (1960)
332. Ludwig, M., Reitzner, M.: A classification of $sl(n)$ invariant values. Ann. Math. (2010). (to appear)
333. Ma, F., Wei, M.S., Mills, W.H.: Correlation structuring and the statistical analysis of steady–state groundwater flow. SIAM J. Sci. Statist. Comput. **8**, 848–867 (1987)
334. Maier, R., Mayer, J., Schmidt, V.: Distributional properties of the typical cell of stationary iterated tessellations. Math. Methods Oper. Res. **59**, 287–302 (2004)
335. Maier, R., Schmidt, V.: Stationary iterated tessellations. Adv. in Appl. Probab. **35**, 337–353 (2003)
336. Malyshev, V.A., Minlos, R.A.: Gibbs Random Fields. Kluwer, Dordrecht (1991)
337. Mandelbrot, B.B., Van Ness, J.: Fractional Brownian motion, fractional noises and applications. SIAM Rev. **10**, 422–437 (1968)
338. Martikaĭnen, A.I.: A converse to the law of the iterated logarithm for a random walk. Theory Probab. Appl. **25**, 361–362 (1980)
339. Matern, B.: Spatial Variation, 2nd edn. Springer, Berlin (1986)
340. Mateu, J., Porcu, E., Gregori, P.: Recent advances to model anisotropic space-time data. Statist. Meth. Applic. **17**, 209–223 (2008)
341. Matheron, G.: Random Sets and Integral Geometry. Wiley, New York (1975)
342. Matthes, K., Kerstan, J., Mecke, J.: Infinitely Divisible Point Processes. Wiley, Chichester (1978)
343. Mayer, M., Molchanov, I.: Limit theorems for the diameter of a random sample in the unit ball. Extremes **10**, 129–150 (2007)
344. Mecke, J.: Stationäre zufällige Maße auf lokalkompakten Abelschen Gruppen. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **9**, 36–58 (1967)
345. Mecke, J.: Parametric representation of mean values for stationary random mosaics. Math. Operationsforsch. Statist. Ser. Statist. **15**, 437–442 (1984)
346. Mecke, J.: Random tessellations generated by hyperplanes, in: Ambartzumian, R. V., Weil, W. (eds.) *Stochastic Geometry, Geometric Statistics, Stereology (Proc. Conf. Oberwolfach, 1983).* Teubner, Leipzig (1984)
347. Mecke, J.: On the relationship between the 0-cell and the typical cell of a stationary random tessellation. Pattern Recognition **32**, 1645–1648 (1999)
348. Meschenmoser, D., Shashkin, A.: Functional central limit theorem for the measure of level sets generated by a Gaussian random field. Statist. Probab. Lett. **81**, 642–646 (2011)
349. Meschenmoser, D., Shashkin, A.: Functional central limit theorem for the measure of level sets generated by a Gaussian random field. Theory Probab. Appl. (2012). (to appear)
350. Miles, R.E.: Random polygons determined by random lines in a plane. Proc. Nat. Acad. Sci. U.S.A. **52**, 901–907 (1964)
351. Miles, R.E.: Random polygons determined by random lines in a plane II. Proc. Nat. Acad. Sci. U.S.A. **52**, 1157–1160 (1964)
352. Miles, R.E.: On the homogeneous planar Poisson point process. Mathematical Biosciences **6**, 85–127 (1970)

353. Miles, R.E.: The various aggregates of random polygons determined by random lines in a plane. Adv. Math. **10**, 256–290 (1973)
354. Miles, R.E.: Some integral geometric formula, with stochastic applications. J. Appl. Probab. **16**, 592–606 (1979)
355. Miles, R.E., Davy, P.J.: Precise and general conditions for the validity of a comprehensive set of stereological fundamental formulae. J. Microsc. **107**, 211–226 (1976)
356. Milman, V.D., Pajor, A.: Isotropic position and inertia ellipsoids and zonoids of the unit ball of a normed *n*-dimensional space, in: *Geometric Aspects of Functional Analysis*, *Lecture Notes in Math.*, vol. 1376. Springer, Berlin (1989)
357. Molchanov, I.: On the convergence of random processes generated by polyhedral approximations of compact convex sets. Theory Probab. Appl. **40**, 383–390 (1996)
358. Molchanov, I.: Theory of Random Sets. Springer, London (2005)
359. Molchanov, I.: Convex geometry of max-stable distributions. Extremes **11**, 235–259 (2008)
360. Molchanov, I.: Convex and star-shaped sets associated with multivariate stable distributions. I. Moments and densities. J. Multivariate Anal. **100**, 2195–2213 (2009)
361. Molchanov, I.S.: Statistics of the Boolean Model for Practitioners and Mathematicians. Wiley, Chichester (1997)
362. Møller, J.: Random Johnson-Mehl tessellations. Adv. in Appl. Probab. **24**, 814–844 (1992)
363. Møller, J.: Lectures on random Voronoi tessellations, in: *Lecture Notes in Statistics* **87**. Springer, New York (1994)
364. Møller, J., Waagepetersen, R.P.: Statistical Inference and Simulation for Spatial Point Processes. Chapman & Hall, Boca Raton (2004)
365. Møller, J., Zuyev, S.: Gamma-type results and other related properties of Poisson processes. Adv. in Appl. Probab. **28**, 662–673 (1996)
366. Móricz, F.: A general moment inequality for the maximum of the rectangular partial sums of multiple series. Acta Math. Hungar. **41**, 337–346 (1983)
367. Mourier, E.: *L*-random elements and L^*-random elements in Banach spaces. In: Proc. Third Berekeley Symp. Math. Statist. and Probability, vol. 2, pp. 231–242. Univ. of California Press (1955)
368. Moussa, A., Sbihi, A., Postaire, J.G.: A Markov random field model for mode detection in cluster analysis. Pattern Recognition Letters **29**, 1197–1207 (2008)
369. Müller, A., Stoyan, D.: Comparison methods for stochastic models and risks. Wiley, Chichester (2002)
370. Nagaev, A.V.: Some properties of convex hulls generated by homogeneous Poisson point processes in an unbounded convex domain. Ann. Inst. Statist. Math. **47**, 21–29 (1995)
371. Nagel, W., Weiss, V.: Crack STIT tessellations: characterization of stationary random tessellations stable with respect to iteration. Adv. in Appl. Probab. **37**, 859–883 (2005)
372. Neveu, J.: Sur les mesures de Palm de deux processus ponctuels stationnaires. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **34**, 199–203 (1976)
373. Newman, C.M.: Normal fluctuations and the FKG inequalities. Comm. Math. Phys. **74**, 119–128 (1980)
374. Newman, C.M.: Asymptotic independence and limit theorems for positively and negatively dependent random variables, in: Tong, Y. L. (ed.) *Inequalities in Statistics and Probability*. Inst. Math. Statist., Hayward, CA (1984)
375. Newman, C.M., Wright, A.L.: An invariance principle for certain dependent sequences. Ann. Probab. **9**, 671–675 (1981)
376. Newman, C.M., Wright, A.L.: Associated random variables and martingale inequalities. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **59**, 361–371 (1982)
377. Nguyen, X.X., Zessin, H.: Punktprozesse mit wechselwirkung. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **37**, 91–126 (1976)
378. Nilsson, M., Kleijn, W.B.: Shannon entropy estimation based on high-rate quantization theory. Proc. XII European Signal Processing Conf. (EUSIPCO) pp. 1753–1756 (2004)
379. Nilsson, M., Kleijn, W.B.: On the estimation of differential entropy from data located on embedded manifolds. IEEE Trans. Inform. Theory **53**, 2330–2341 (2007)

380. Norberg, T.: Existence theorems for measures on continuous posets, with applications to random set theory. Math. Scand. **64**, 15–51 (1989)
381. Ohser, J., Stoyan, D.: On the second-order and orientation analysis of planar stationary point processes. Biom. J. **23**, 523–533 (1981)
382. Okabe, A., Boots, B., Sugihara, K., Chiu, S.N.: Spatial Tessellations: Concepts and Applications of Voronoi Diagrams. With a foreword by D. G. Kendall. Wiley, Chichester (2000)
383. Pantle, U., Schmidt, V., Spodarev, E.: Central limit theorems for functionals of stationary germ–grain models. Ann. Appl. Probab. **38**, 76–94 (2006)
384. Pantle, U., Schmidt, V., Spodarev, E.: On the estimation of integrated covariance functions of stationary random fields. Scand. J. Statist. **37**, 47–66 (2010)
385. Paranjape, S.R., Park, C.: Laws of iterated logarithm of multiparameter Wiener processes. J. Multivariate Anal. **3**, 132–136 (1973)
386. Paroux, K.: Quelques théorèmes centraux limites pour les processus Poissoniens de droites dans le plan. Adv. in Appl. Probab. **30**, 640–656 (1998)
387. Paulauskas, V.: On Beveridge–Nelson decomposition and limit theorems for linear random fields. J. Multivar. Anal. **101**, 621–639 (2010)
388. Paulauskas, V.: On Beveridge-Nelson decomposition and limit theorems for linear random fields. J. Multivariate Anal. **101**, 621–639 (2010)
389. Peligrad, M.: Maximum of partial sums and an invariance principle for a class of weak dependent random variables. Proc. Amer. Math. Soc. **126**, 1181–1189 (1998)
390. Penrose, M.: Random Geometric Graphs. Oxford Univ. Press (2003)
391. Penrose, M.D.: Gaussian limits for random geometric measures. Electron. J. Probab. **12**, 989–1035 (2007)
392. Penrose, M.D.: Laws of large numbers in stochastic geometry with statistical applications. Bernoulli **13**, 1124–1150 (2007)
393. Penrose, M.D., Wade, A.R.: Multivariate normal approximation in geometric probability. J. Stat. Theory Pract. **2**, 293–326 (2008)
394. Penrose, M.D., Yukich, J.E.: Central limit theorems for some graphs in computational geometry. Ann. Appl. Probab. **11**, 1005–1041 (2001)
395. Penrose, M.D., Yukich, J.E.: Mathematics of random growing interfaces. J. Phys. A **34**, 6239–6247 (2001)
396. Penrose, M.D., Yukich, J.E.: Limit theory for random sequential packing and deposition. Ann. Appl. Probab. **12**, 272–301 (2002)
397. Penrose, M.D., Yukich, J.E.: Weak laws of large numbers in geometric probability. Ann. Appl. Probab. **13**, 277–303 (2003)
398. Penrose, M.D., Yukich, J.E.: Normal approximation in geometric probability, in: Barbour, A. D., Chen, L. H. Y. (eds.) *Stein's Method and Applications*. Lecture Note Series, vol. 5. Institute for Mathematical Sciences, National University of Singapore (2005)
399. Penrose, M.D., Yukich, J.E.: Limit theory for point processes on manifolds (2010). ArXiv:1104.0914
400. Petkantschin, B.: Integralgeometrie 6. Zusammenhänge zwischen den dichten der linearen Unterräume im *n*-dimensionalen Raum. Abf. Math. Sem. Univ. Hamburg **11**, 249–310 (1936)
401. Petrov, V.V.: Limit Theorems of Probability Theory. Clarendon Press, Oxford (1995)
402. Pitt, L.D.: Positively correlated normal variables are associated. Ann. Probab. **10**, 496–499 (1982)
403. Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P.: Numerical Recipes in C. The Art of Scientific Computing, 2nd edn. Cambridge Univ. Press, Cambridge (1997)
404. Preston, C.J.: Gibbs States on Countable Sets. Cambridge Univ. Press (1974)
405. Pruitt, W.E.: General one-sided laws of the iterated logarithm. Ann. Probab. **9**, 1–48 (1981)
406. Quine, M.P., Watson, D.F.: Radial generation of *n*-dimensional Poisson processes. J. Appl. Probab. **21**, 548–557 (1984)
407. Quintanilla, J., Torquato, S.: Local volume fluctuations in random media. J. Chem. Phys. **106**, 2741–2751 (1997)
408. Rachev, S.T., Mittnik, S.: Stable Paretian Models in Finance. Wiley, New York (2000)

409. Račkauskas, A., Suquet, C., Zemlys, V.: Hölderian functional central limit theorem for multi–indexed summation process. Stochastic Process. Appl. **117**(8), 1137–1164 (2007)
410. Rajput, B.S., Rosinski, J.: Spectral representations of infinitely divisible processes. Probab. Th. Rel. Fields **82**, 451–487 (1989)
411. Raynaud, H.: Sur l'enveloppe convexe des nuages de points aléatoires dans R^n. I. J. Appl. Probab. **7**, 35–48 (1970)
412. Redmond, C.: Boundary rooted graphs and Euclidean matching algorithms. Ph.D. thesis, Department of Mathematics, Lehigh University, Bethlehem, PA (1993)
413. Redmond, C., Yukich, J.E.: Limit theorems and rates of convergence for subadditive Euclidean functionals. Ann. Appl. Probab. **4**, 1057–1073 (1994)
414. Redmond, C., Yukich, J.E.: Limit theorems for Euclidean functionals with power-weighted edges. Stochastic Process. Appl. **61**, 289–304 (1996)
415. Reitzner, M.: Random polytopes and the Efron-Stein jackknife inequality. Ann. Probab. **31**, 2136–2166 (2003)
416. Reitzner, M.: Random polytopes and the Efron-Stein jackknife inequality. Ann. Probab. **31**, 2136–2166 (2003)
417. Reitzner, M.: Central limit theorems for random polytopes. Probab. Theory Related Fields **133**, 483–507 (2005)
418. Reitzner, M.: Random polytopes, in: Kendall, W. S., Molchanov, I. (eds.) *New Perspectives in Stochastic Geometry*. Oxford Univ. Press. (2010)
419. Rényi, A.: On a one-dimensional random space-filling problem. MTA Mat Kut. Int. Ǩol. (Publications of the Math. Res. Inst. of the Hungarian Academy of Sciences) **3**, 109–127 (1958)
420. Rényi, A., Sulanke, R.: Über die konvexe Hülle von n zufällig gewählten Punkten. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **2**, 75–84 (1963)
421. Rényi, A., Sulanke, R.: Über die konvexe Hülle von n zufällig gewählten Punkten. II. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **3**, 138–147 (1964)
422. Resnick, S.: Extreme Values, Regular Variation and Point Processes. Springer, New York (2008)
423. Ripley, B.D.: The second-order analysis of stationary point processes. J. Appl. Probab. **13**, 255–266 (1976)
424. Rosalsky, A.: On the converse to the iterated logarithm law. Sankhyā **42**, 103–108 (1980)
425. Rosenblatt, M.: Stationary sequences and random fields. Birkhäuser, Boston (1985)
426. Rother, W., Zähle, M.: A short proof of the principal kinematic formula and extensions. Trans. Amer. Math. Soc. **321**, 547–558 (1990)
427. Roweis, S., Saul, L.: Nonlinear dimensionality reduction by locally linear embedding. Science **290** (2000)
428. Roy, P.: Ergodic theory, Abelian groups and point processes induced by stable random fields. Ann. Probab. **38**, 770–793 (2010)
429. Rozanov, Y.A.: Markov Random Fields. Springer, New York (1982)
430. Rue, H., Held, L.: Gaussian Markov Random Fields: Theory and Applications. Chapman & Hall, Boca Raton (2005)
431. Ruelle, D.: Statistical Mechanics: Rigorous Results. World Scientific, Singapore (1969)
432. Samorodnitsky, G.: Long Range Dependence. Now Publishers, Delft (2007)
433. Samorodnitsky, G., Taqqu, M.: Stable non-Gaussian random processes. Stochastic Models with infinite variance. Chapman & Hall, Boca Raton (1994)
434. Santaló, L.A.: Integral Geometry and Geometric Probability. Addison-Wesley, Reading, MA (1976)
435. Sarkar, T.K.: Some lower bounds of reliability. Tech. Rep. 124, Dept. of Operation Research and Statistics, Stanford University (1969)
436. Sasvári, A.: Positive Definite and Definitizable Functions. Akademie-Verlag (1994)
437. Sato, K.I.: Lévy Processes and Infinitely Divisible Distributions. Cambridge Univ. Press, Cambridge (1999)
438. Schlather, M.: Introduction to positive definite functions and to unconditional simulation of random fields. Tech. rep., Lancaster University (1999)

439. Schlather, M.: On the second-order characteristics of marked point processes. Bernoulli **7**, 99–117 (2001)
440. Schlather, M.: Models for stationary max-stable random fields. Extremes **5**, 33–44 (2002)
441. Schlather, M.: Construction of positive definite functions. Lecture at $15^t h$ Workshop on Stoch. Geometry, Stereology and Image Analysis, Blaubeuren (2009)
442. Schlather, M.: Some covariance models based on normal scale mixtures. Bernoulli **16**, 780–797 (2010)
443. Schneider, R.: Approximation of convex bodies by random polytopes. Aequationes Math. **32**, 304–310 (1987)
444. Schneider, R.: Random approximation of convex sets. J. Microsc. **151**, 211–227 (1988)
445. Schneider, R.: Convex bodies: the Brunn-Minkowski theory. Cambridge Univ. Press, Cambridge (1993)
446. Schneider, R.: Discrete aspects of stochastic geometry, in: Goodman, J. E., O'Rourke, J. (eds.) *Handbook of Discrete and Computational Geometry*. CRC Press, Boca Raton, FL (1997)
447. Schneider, R.: Recent results on random polytopes. Boll. Unione Mat. Ital. **1**, 17–39 (2008)
448. Schneider, R.: Weighted faces of Poisson hyperplane tessellations. Adv. in Appl. Probab. **41**, 682–694 (2009)
449. Schneider, R.: Extremal properties of random mosaics, in: Bárány, I., Böröczky, K. J., Fejes Tóth, G., Pach, J. (eds.) *Geometry - Intuitive, Discrete, and Convex*. Springer (2010). (to appear)
450. Schneider, R.: Vertex numbers of weighted faces in Poisson hyperplane mosaics. Discrete Comput. Geom. **44**, 599–607 (2010)
451. Schneider, R., Weil, W.: Translative and kinematic integral formulae for curvature measures. Math. Nachr. **129**, 67–80 (1986)
452. Schneider, R., Weil, W.: Stochastic and Integral Geometry. Springer, Berlin (2008)
453. Schneider, R., Wieacker, J.A.: Random polytopes in a convex body. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **52**, 69–73 (1980)
454. Schoenberg, I.J.: Metric spaces and complete monotone functions. Ann. of Math **39**, 811–841 (1938)
455. Schoutens, W.: Lévy processes in finance: pricing financial derivatives. Wiley, Chichester (2003)
456. Schreiber, T.: Variance asymptotics and central limit theorems for volumes of unions of random closed sets. Adv. in Appl. Probab. **34**, 520–539 (2002)
457. Schreiber, T.: Asymptotic geometry of high density smooth-grained Boolean models in bounded domains. Adv. in Appl. Probab. **35**, 913–936 (2003)
458. Schreiber, T.: (2009). Personal communication
459. Schreiber, T.: Limit theorems in stochastic geometry, in: Kendall, W. S., Molchanov. I. (eds.) *New Perspectives in Stochastic Geometry*. Oxford Univ. Press. (2010)
460. Schreiber, T., Penrose, M.D., Yukich, J.E.: Gaussian limits for multidimensional random sequential packing at saturation. Comm. Math. Phys. **272**, 167–183 (2007)
461. Schreiber, T., Yukich, J.E.: Large deviations for functionals of spatial point processes with applications to random packing and spatial graphs. Stochastic Process. Appl. **115**, 1332–1356 (2005)
462. Schreiber, T., Yukich, J.E.: Stabilization and limit theorems for geometric functionals of Gibbs point processes (2008). URL http://arxiv:0802.0647v1
463. Schreiber, T., Yukich, J.E.: Variance asymptotics and central limit theorems for generalized growth processes with applications to convex hulls and maximal points. Ann. Probab. **36**, 363–396 (2008)
464. Schütt, C.: Random polytopes and affine surface area. Math. Nachr. **170**, 227–249 (1994)
465. Seneta, E.: Regularly Varying Functions. Springer, Berlin (1976)
466. Seppäläinen, T., Yukich, J.E.: Large deviation principles for Euclidean functionals and other nearly additive processes. Prob. Theory Relat. Fields **120**, 309–345 (2001)
467. Shao, Q.M.: A comparison theorem on moment inequalities between negatively associated and independent random variables. J. Theoret. Probab. **13**, 343–356 (2000)

468. Shashkin, A.: A strong invariance principle for positively or negatively associated random fields. Statist. Probab. Lett. **78**, 2121–2129 (2008)
469. Shashkin, A.P.: Quasi-associatedness of a Gaussian system of random vectors. Russ. Math. Surv. **57**, 1243–1244 (2002)
470. Shashkin, A.P.: A dependence property of a spin system. In: Transactions of XXIV Int. Sem. on Stability Problems for Stoch. Models, pp. 30–35. Yurmala, Latvia (2004)
471. Shashkin, A.P.: Maximal inequality for a weakly dependent random field. Math. Notes **75**, 717–725 (2004)
472. Shashkin, A.P.: On the Newman central limit theorem. Theory Probab. Appl. **50**, 330–337 (2006)
473. Shepp, L.: Covering the circle with random arcs. Israel J. Math. **11**, 328–345 (1972)
474. Shinozuka, M., Jan, C.M.: Digital simulation of random processes and its applications. J. Sound Vibration **25**, 111–128 (1972)
475. Shiryaev, A.N.: Probability, *Graduate Texts in Mathematics*, vol. 95, 2nd edn. Springer, New York (1996)
476. Shiryaev, A.N.: Essentials of Stochastic Finance. Facts, Models, Theory. World Scientific, River Edge, NJ (1999)
477. Shorack, G.R., Smythe, R.T.: Inequalities for $\max |S_{\mathbf{k}}|/b_{\mathbf{k}}$ where $\mathbf{k} \in N^r$. Proc. Amer. Math. Soc. **54**, 331–336 (1976)
478. Siegel, A.F., Holst, L.: Covering the circle with random arcs of random sizes. J. Appl. Probab. **19**, 373–381 (1982)
479. Smythe, R.T.: Strong laws of large numbers for r-dimensional arrays of random variables. Ann. Probab. **1**, 164–170 (1973)
480. Smythe, R.T.: Sums of independent random variables on partially ordered sets. Ann. Probab. **2**, 906–917 (1974)
481. Speed, T.P., Kiiveri, H.T.: Gaussian Markov distributions over finite graphs. Ann. Statist. **14**, 138–150 (1986)
482. Stadtmüller, U., Thalmaier, M.: Strong laws for delayed sums of random fields. Acta Sci. Math. (Szeged) **75**, 723–737 (2009)
483. Stanley, R.P.: Enumerative combinatorics. Vol. 2, *Cambridge Studies in Advanced Mathematics*, vol. 62. Cambridge Univ. Press, Cambridge (1999). With a foreword by Gian-Carlo Rota and appendix 1 by Sergey Fomin
484. Steele, J.M.: Subadditive Euclidean functionals and nonlinear growth in geometric probability. Prob. Theory Relat. Fields **9**, 365–376 (1981)
485. Steele, J.M.: Probability Theory and Combinatorial Optimization. SIAM (1997)
486. Steinebach, J.: On the increments of partial sum processes with multidimensional indices. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **63**, 59–70 (1983)
487. Sterio, D.C.: The unbiased estimation of number and sizes of aritrary particles using the disector. J. Microsc. **134**, 127–136 (1984)
488. Stevens, W.L.: Solution to a geometrical problem in probability. Ann. Eugenics **9**, 315–320 (1939)
489. Stoev, S.A.: On the ergodicity and mixing of max-stable processes. Stochastic Process. Appl. **118**, 1679–1705 (2008)
490. Stoev, S.A., Taqqu, M.S.: Extremal stochastic integrals: a parallel between max-stable processes and α-stable processes. Extremes **8**, 237–266 (2006)
491. Stoyan, D.: Fundamentals of point process statistics, in: Baddeley, A., Gregori, P., Mateu, J., Stoica, R., Stoyan, D. (eds.) *Case Studies in Spatial Point Process Modeling*. Lecture Notes in Statistics **185**. Springer, New York (2006)
492. Stoyan, D., Kendall, W., Mecke, J.: Stochastic Geometry and its Applications, 2nd edn. Wiley (1995)
493. Stoyan, D., Stoyan, H., Jansen, U.: Umweltstatistik. Teubner, Stuttgart (1997)
494. Strassen, V.: A converse to the law of the iterated logarithm. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **4**, 265–268 (1966)
495. Stroock, D.: Probability Theory, an Analytic View. Cambridge Univ. Press, Cambridge (1993)

496. Tchoumatchenko, K., Zuyev, S.: Aggregate and fractal tessellations. Probab. Theory Related Fields **121**, 198–218 (2001)
497. Tenenbaum, J.B., de Silva, V., Langford, J.C.: A global geometric framework for nonlinear dimensionality reduction. Science **290**, 2319B–2323 (2000)
498. Thorisson, H.: Coupling, Stationarity and Regeneration. Springer, New York (2000)
499. Tone, C.: A central limit theorem for multivariate strongly mixing random fields. Probab. Math. Statist. **30**(2), 215–222 (2010)
500. Torquato, S.: Random Heterogeneous Materials. Springer, New York (2002)
501. Tyurin, I.S.: Sharpening the upper bounds for constants in Lyapunov's theorem. Russian Math. Surveys **65**, 586–588 (2010)
502. Uchaikin, V.V., Zolotarev, V.M.: Chance and Stability. Stable Distributions and their Applications. VSP, Utrecht (1999)
503. Véhel, J.L.: Fractals in Engineering: From Theory to Industrial Applications. Springer, New York (1997)
504. Voss, F., Gloaguen, C., Fleischer, F., Schmidt, V.: Densities of shortest path lengths in spatial stochastic networks (Preprint) (2009)
505. Voss, F., Gloaguen, C., Fleischer, F., Schmidt, V.: Distributional properties of Euclidean distances in wireless networks involving road systems. IEEE Journal on Selected Areas in Communications **27**, 1047–1055 (2009)
506. Voss, F., Gloaguen, C., Schmidt, V.: Scaling limits for shortest path lengths along the edges of stationary tessellations (Preprint) (2009)
507. Vronski, M.A.: Rate of convergence in the SLLN for associated sequences and fields. Theory Probab. Appl. **43**, 449–462 (1999)
508. Vu, V.: Sharp concentration of random polytopes. Geom. Funct. Anal. **15**, 1284–1318 (2005)
509. Vu, V.: Central limit theorems for random polytopes in a smooth convex set. Adv. Math. **207**, 221–243 (2006)
510. Wackernagel, H.: Multivariate Geostatistics. An Introduction with Applications, 3rd edn. Springer, Berlin (2003)
511. Walsh, J.B.: An introduction to stochastic partial differential equations. In: École d'été de probabilités de Saint-Flour, XIV—1984, *Lecture Notes in Math.*, vol. 1180, pp. 265–439. Springer, Berlin (1986)
512. Wang, Y., Stoev, S.A., Roy, P.: Ergodic properties of sum- and max- stable stationary random fields via null and positive group actions. Preprint (2009). ArXiv:0911.0610
513. Watkins, K.P., Hickman, A.H.: Geological evolution and mineralization of the Murchison Province, Western Australia. Bulletin 137, Geological Survey of Western Australia (1990). Published by Department of Mines, Western Australia. Available online from Department of Industry and Resources, State Government of Western Australia, www.doir.wa.gov.au
514. Weil, W.: An application of the central limit theorem for Banach-space-valued random variables to the theory of random sets. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **60**, 203–208 (1982)
515. Weil, W.: Densities of quermassintegrals for stationary random sets, in: Ambartzumian, R. V., Weil, W. (eds.) *Stochastic Geometry, Geometric Statistics, Stereology (Proc. Conf. Oberwolfach, 1983)*. Teubner, Leipzig (1984)
516. Weil, W.: Densities of mixed volumes for Boolean models. Adv. in Appl. Prob. **33**, 39–60 (2001)
517. Weil, W.: Mixed measures and functionals of translative integral geometry. Math. Nachr. **223**, 161–184 (2001)
518. Weil, W., Wieacker, J.A.: Densities for stationary random sets and point processes. Adv. in Appl. Prob. **16**, 324–346 (1984)
519. Weil, W., Wieacker, J.A.: Stochastic geometry, in: Gruber, P. M., Wills, J. M. (eds.) *Handbook of Convex Geometry, vol. B*. North-Holland, Amsterdam (1993)
520. van de Weygaert, R.: Fragmenting the universe III. The construction and statistics of 3-D Voronoi tessellations. Astron. Astrophys. **283**, 361–406 (1994)
521. Wichura, M.J.: Some Strassen-type laws of the iterated logarithm for multiparameter stochastic processes with independent increments. Ann. Probab. **1**, 272–296 (1973)

522. Wieacker, J.A.: Einige Probleme der polyedrischen Approximation. Diplomarbeit, Albert-Ludwigs-Universität, Freiburg i. Br. (1978)
523. Willinger, W., Paxson, V., Taqqu, M.S.: Self-similarity and heavy tails: Structural modeling of network traffic. In: A Practical Guide to Heavy Tails, pp. 27–53. Birkhäuser, Boston, MA, (1998)
524. Winkler, G.: Image Analysis, Random Fields and Markov Chain Monte Carlo Methods, 2nd edn. Springer, Berlin (2003)
525. Wood, A.T.A., Chan, G.: Simulation of stationary Gaussian processes in $[0,1]^d$. J. Comput. Graph. Statist. **3**, 409–432 (1994)
526. Yoshihara, K.: Limiting behavior of U-statistics for stationary, absolutely regular processes. Z. Wahrscheinlichkeitstheorie und verw. Gebiete **35**, 237–252 (1976)
527. Yuan, J., Subba Rao, T.: Higher order spectral estimation for random fields. Multidimens. Syst. Signal Process. **4**, 7–22 (1993)
528. Yukich, J.E.: Probability Theory of Classical Euclidean Optimization Problems. Lecture Notes in Mathematics **1675**. Springer, Berlin (1998)
529. Yukich, J.E.: Limit theorems for multi-dimensional random quantizers. Electron. Comm. Probab. **13**, 507–517 (2008)
530. Yukich, J.E.: Point process stabilization methods and dimension estimation. Proceedings of Fifth Colloquium of Mathematics and Computer Science. Discrete Math. Theor. Comput. Sci. pp. 59–70 (2008)
531. Zähle, M.: Curvature measures and random sets I. Math. Nachr. **119**, 327–339 (1984)
532. Zhang, L.X., Wen, J.: A weak convergence for functions of negatively associated random variables. J. Multivariate Anal. **78**, 272–298 (2001)
533. Ziegler, G.: Lectures on 0/1 polytopes, in: Kalai, G., Ziegler, G. M. (eds.) *Polytopes – Combinatorics and Computation, DMV Seminars*. Birkhäuser (2000)
534. Zolotarev, V.M.: Modern Theory of Summation of Random Variables. VSP, Utrecht (1997)
535. Zuker, M., Mathews, D.H., Turner, D.H.: Algorithms and thermodynamics for RNA secondary structure prediction: A practical guide, in: Barciszewski, J., Clark, B. F. C. (eds.) *RNA Biochemistry and Biotechnology*. NATO ASI Series. Kluwer, Dordrecht (1999)
536. Zuyev, S.: Strong Markov property of Poisson processes and Slivnyak formula. Lecture Notes in Statistics. **185**. Springer (2006)
537. Zuyev, S.: Stochastic geometry and telecommunication networks, in: Kendall, W. S. and Molchanov, I. (eds.) *New Perspectives in Stochastic Geometry*. Oxford Univ. Press. (2010)