Anomalous correlations in the simple exclusion process with reservoirs

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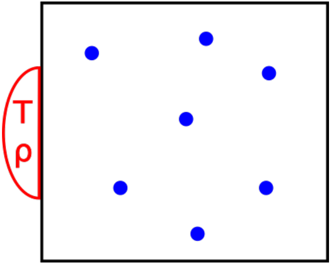
Rencontres de probabilités 2021
22 Octobre 2021

Motivations



Objective:

Quantify the probability of observing anomalous macroscopic correlations in long time.

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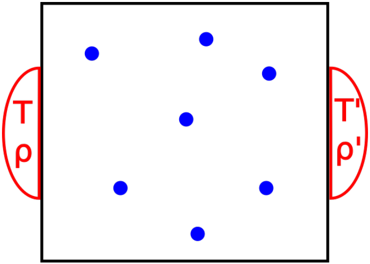

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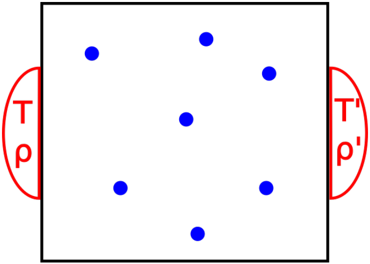

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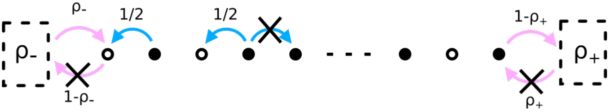
The framework



ν_{ss}^N: stationary state of the open SSEP on $\{-(N-1), ..., N-1\}$ with reservoirs at densities $0 < \rho_- \leq \rho_+ < 1$.
State space: $\Omega_N := \{0,1\}^{2N-1}$. $\eta \in \Omega_N$: a configuration.

- When $\rho_- = \rho_+ = \rho \in [0,1]$: reversible dynamics, no long-time correlations:

$$\nu_{ss}^N = \bigotimes_{|i|\leq N-1} \mathrm{Ber}(\rho), \qquad \mathrm{Ber}(\rho)(\{1\}) = \rho = 1 - \mathrm{Ber}(\rho)(\{0\}).$$

Correlations in the stationary state

- When $\rho_- \neq \rho_+$, stationary density profile $\bar\rho$ solves:

$$\Delta\bar\rho = 0, \quad \bar\rho(\pm 1) = \rho_\pm.$$

$$\forall x \in (-1,1), \qquad \lim_{N\to\infty} \nu_{ss}^N(\eta_{\lfloor xN\rfloor}) = \bar\rho(x) = \frac{(\rho_+ - \rho_-)x}{2} + \frac{\rho_+ + \rho_-}{2}.$$

- Existence of a macroscopic current of particles, inducing long range correlations [Spo83][DLS02]: if $x \neq y \in (-1,1)$,

$$\lim_{N\to\infty} NE_{\nu_{ss}^N}\left[(\eta_{\lfloor xN\rfloor} - \bar\rho_x)(\eta_{\lfloor yN\rfloor} - \bar\rho_y)\right] = (\bar\rho')^2\Delta_{1d}^{-1}(x,y) =: k_0(x,y).$$

with Δ_{1d}^{-1} the kernel of the inverse one-dimensional Dirichlet Laplacian.

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Statement of the problem

Question:

What is the probability that, in long time and for N large, the time averaged correlations differ from those of the stationary state?

- Formalisation: for a function $\phi : [-1,1]^2 \to \mathbb{R}$, the two-point correlation field $\Pi^N(\phi)$ is given by:

$$\Pi^N(\phi) := \frac{1}{4N} \sum_{\substack{|i|,|j| \leq N-1 \\ i \neq j}} \bar{\eta}_i \bar{\eta}_j \phi_{i,j}, \quad \bar{\eta}_i := \eta_i - \bar{\rho}(i/N), \quad \phi_{i,j} = \phi\Big(\frac{i}{N}, \frac{j}{N}\Big).$$

Reformulated question:

Estimate, for a correlation kernel k and when $N \gg 1$ and $T \gg 1$:

$$\mathbb{P}^N_{\rho_\pm}\Big(\frac{1}{T}\int_0^T \Pi^N_t(\phi)dt \approx \int_{(-1,1)^2} k(x,y)\phi(x,y)dxdy\Big).$$

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Long time, fixed N behaviour

Theorem (Donsker, Varadhan 1975)

$N \in \mathbb{N}^*$ fixed. μ^N: probability measure on $\Omega_N = \{0,1\}^{2N-1}$.

$$\lim_{T\to\infty} \frac{1}{T} \log \mathbb{P}^N_{\rho_\pm}\Big(\frac{1}{T}\int_0^T \delta_{\eta_t=\eta} dt \approx \mu^N\Big) = -I^N_{DV}(\mu^N),$$

with $I^N_{DV}(\mu^N)$ solving a variational problem.

- Reversible case $(\rho_+ = \rho_- = \rho \in (0,1))$:

$$\nu^N_{ss} = \nu^N_\rho := \bigotimes_{|i|\le N-1} \mathrm{Ber}(\rho), \quad \mathrm{Ber}(\rho)(\{1\}) = \rho = 1 - \mathrm{Ber}(\rho)(\{0\}),$$

and I^N_{DV} becomes:

$$I^N_{DV}(\mu^N) = D^N_\rho(f^{1/2}_\mu) := -\nu^N_\rho(f^{1/2}_\mu L_\rho f^{1/2}_\mu), \qquad f_\mu = \frac{d\mu^N}{d\nu^N_\rho}.$$

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Non reversible case

Problem 1:

If $\rho_+ \neq \rho_-$, Donsker and Varadhan's variational formula does not have simple solutions.

Probleme 2:

Even in the reversible case, the probabilities of observing anomalous density or correlation profiles *do not live at the same scale*:

$$\mathbb{P}^N_{\text{rev}}\left(\left|\frac{1}{TN}\int_0^T \sum_{|i|\leq N} \eta_i(t) f_i dt - \rho \int_{-1}^1 f(x)dx\right| > \varepsilon\right) \overset{T\gg 1}{\approx} e^{-c(\varepsilon)TN^{-1}},$$

$$\mathbb{P}^N_{\text{rev}}\left(\left|\frac{1}{T}\int_0^T \Pi^N_t(\phi)dt\right| > \varepsilon\right) \overset{T\gg 1}{\approx} e^{-c'(\varepsilon)TN^{-2}}.$$

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Summary

Objective:

Estimate, for a kernel k and a test function ϕ, in the $N, T \gg 1$ limits:

$$\mathbb{P}^N_{\rho_\pm}\left(\frac{1}{T}\int_0^T \Pi^N_t(\phi)dt \approx \int_{\boxslash} k(x,y)\phi(x,y)dxdy\right),$$

with $\boxslash := (-1,1)^2 \setminus \{(x,x) : x \in (-1,1)\}$.

Approach:

- Consider $\frac{1}{T}\int_0^T \Pi^N_{tN^2} dt$ (diffusive scale) for $N \gg 1$ with T fixed. Advantage: tools from hydrodynamic limits available.
- [KOV89]: dynamics is tilted by well-chosen biases, of type $\Pi^N(h)$ for a test function h. Equations then need to be closed.
- Key ingredient to close the equations and obtain quantitative long-time estimates: the relative entropy method.

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The large deviations

$\mathcal{T}'_s$: symetric distributions in $\left(\mathbb{H}^2(\boxslash)\right)'$. Equipped with the weak-star topology.

Theorem

Let $0 < \rho_- < \rho_+ < 1$ be sufficiently close to $1/2$.

There is a functional $I : \mathcal{T}'_s \to \bar{\mathbb{R}}_+$, such that $I = \infty$ outside of $\mathbb{H}^1(\boxslash)$, and for any compact set $\mathcal{K}$:

$$\limsup_{T\to\infty} \limsup_{N\to\infty} \frac{1}{T} \log \mathbb{P}^N_{\rho_\pm}\left(\frac{1}{T}\int_0^T \Pi^N_{tN^2}\, dt \in \mathcal{K}\right) \leq -\inf_{\mathcal{K}} I.$$

Let k be a smooth correlation kernel, close enough to the correlations $k_0 = (\bar{\rho}')^2 \Delta^{-1}_{1d}$ in the steady state. Then:

$$\liminf_{T\to\infty} \liminf_{N\to\infty} \frac{1}{T} \log \mathbb{P}^N_{\rho_\pm}\left(\frac{1}{T}\int_0^T \Pi^N_{tN^2}\, dt \approx k(\cdot)\right) \geq -I(k).$$

The relative entropy method

• Due to Yau (1991), used by Bertini, Funaki, Landim, Olla, Quastel, Rezakhanlou, Varadhan... in the 90's. Allows for the study of the typical behaviour of the density in many models.
Refined by Jara and Menezes (2018) to study fluctuations.

• Central idea: find a family $(\mu_t^N)_{t\leq T}$ of measures, both as simple as possible and close to the law $f_{tN^2}\mu_t^N$ of the dynamics on $[0, T]$, $T > 0$, in the sense:

$$H\big(f_{tN^2}\mu_t^N|\mu_t^N\big) := \mu_t^N\big(f_{tN^2}\log f_{tN^2}\big) \text{ is "small" when } N \text{ is large.}$$

• Entropy inequality: for each $V : \Omega_N \to \mathbb{R}$,

$$\mu_t^N\big(f_{tN^2}V\big) \leq H\big(f_{tN^2}\mu_t^N|\mu_t^N\big) + \log \mu_t^N\Big(e^V\Big).$$

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The relative entropy method (2)

Useful orders of magnitude:

$$\sup_{t\leq T} H\big(f_{tN^2}\mu_t^N | \mu_t^N\big) = o(N) \quad \text{for the density,}$$

$$\sup_{t\leq T} H\big(f_{tN^2}\mu_t^N | \mu_t^N\big) = o(N^{1/2}) \quad \text{for fluctuations,}$$

$$\sup_{t\leq T} H\big(f_{tN^2}\mu_t^N | \mu_t^N\big) = o_N(1) \quad \text{for corrélations.}$$

Heuristics: observables

- for the density : $\displaystyle\sum_{-N\leq i\leq N} \eta_i \phi(i/N) \approx O(N),$

- for fluctuations : $\displaystyle\sum_{-N\leq i\leq N} \bar{\eta}_i \phi(i/N) \approx O(N^{1/2}).$

- for correlations : $\displaystyle\frac{1}{N}\sum_{i\neq j} \bar{\eta}_i\bar{\eta}_j \psi(i/N, j/N) \approx O_N(1).$

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Choosing the measures μ_t^N: density and fluctuations

• Intuition: the particle system is *locally at equilibrium*, i.e the dynamics in each small macroscopic box is at equilibrium at the local value of the density, and independent from the rest of the system.

• Consequence: compare the law of the dynamics to an uncorrelated measure with the correct local densities:

$$\mu_t^N = \bigotimes_{-N \leq i \leq N} \mathrm{Ber}\big(\rho(t, x/N)\big),$$

with, for the open SSEP :

$$\partial_t \rho = \Delta \rho, \quad \rho(\cdot, \pm 1) = \rho_\pm, \quad \rho(0, \cdot) = \rho_0.$$

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Theorem (≈ Jara, Menezes 2018)

For the open SSEP : $\sup_{t\leq T} H\big(f_{tN^2}\mu_t^N|\mu_t^N\big) \leq C(T)$ with μ_t^N product.

- The theorem is optimal when μ_t^N is product.
- To study correlations, one needs: $\sup_{t\leq T} H\big(f_{tN^2}\mu_t^N|\mu_t^N\big) = o_N(1)$.

Idea:

Compare to a "gaussian" measure with correlations:

$$\nu_{g_t}^N(\eta) = \frac{1}{\mathcal{Z}_{g_t}^N} \exp\left[\frac{1}{2N}\sum_{i\neq j}\bar{\eta}_i\bar{\eta}_j g_t\Big(\frac{i}{N},\frac{j}{N}\Big)\right]\mu_t^N(\eta), \quad \eta\in\Omega_N.$$

Optimise then $(g_t)_t$ to minimise $H\big(f_{tN^2}\nu_{g_t}^N|\nu_{g_t}^N\big)$.

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The entropy estimate

Theorem

For the SSEP dynamics with reservoirs at sufficiently close density ρ_-, ρ_+, there is a regular function g such that the law $f_t \nu_g^N$ of the above dynamics at time $t \geq 0$ satisfies:

$$H(f_{tN^2}\nu_g^N | \nu_g^N) \leq C(T)\Big(H(f_0 \nu_g^N | \nu_g^N) + N^{-1/2}\Big).$$

- Higher-dimensional entropy estimates?

- Other 1d gradient systems.

- Method allows for the study of correlations/fluctuations conditioned to certain rare events. Example: fluctuations of a SSEP on a ring conditioned to having a macroscopic current.

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