**Uniform ergodicity and the one-sided ergodic Hilbert transform**

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Abstract

A bounded linear operator $T$ on a (real or complex) Banach space $X$ is called *mean ergodic* if its averages $M_n(T)x := \frac{1}{n}\sum_{k=1}^{n} T^k x$ converge in $X$ for every $x \in X$. The operator $T$ is called *uniformly ergodic* if the averages converge in operator norm.

Mean ergodicity implies that $\sup_n \|M_n(T)\| < \infty$ and $\frac{1}{n}T^n x \to 0$ for every $x$, and yields the *ergodic decomposition* $X = \{x \in X : Tx = x\} \oplus \overline{(I-T)X}$. By the Hahn-Banach theorem, $x \in \overline{(I-T)X}$ if and only if $\langle y^*, x\rangle = 0$ whenever $T^*y^* = y^*$. When $T$ is mean ergodic, $M_n(T)x \to 0$ if and only if $x \in \overline{(I-T)X}$.

The *one-sided ergodic Hilbert transform* of $T$ is the (unbouded) operator $Hx := \sum_{n=1}^{\infty} n^{-1}T^n x$, with domain $D(H) = \{x \in X : Hx \text{ exists}\}$. It follows that $D(H) \subset \overline{(I-T)X}$.

It was shown in 1974 that $T$ is uniformly ergodic if and only if $\frac{1}{n}\|T^n\| \to 0$ and $(I-T)X$ is closed. It follows from a result of M.E. Becker (2011) that if $T$ is uniformly ergodic, then $(I-T)X = D(H) = \overline{(I-T)X}$. In this talk we investigate conditions on $D(H)$ which imply uniform ergodicity. We use the results to prove the following theorem Glück (2015):

**Theorem.** *Let $T$ be a bounded linear operator on a complex Banach space $X$. If for every $x \in X$ there is $p \in [1,\infty)$ such that $\sum_{n=1}^{\infty} \|T^n x\|^p < \infty$, then $\|T^n\| \to 0$.*

**Joint work with Guy Cohen**