

Rencontres de Probabilités 2024

Rouen, 26-27 septembre 2024

Programme

Thursday September 26 2024

- 11h - 12:15 mini-course A (Part I), **Gunter Schütz** (Forschungszentrum Jülich)

Title: Duality and symmetry in interacting particle systems

- 12:15 - 14:00 lunch

- 14:00 - 14:30 **Seonwoo Kim** (KIAS, Séoul)

Title: *Metastable Hierarchy in Abstract Low-Temperature Lattice Models*

- 14:30 - 15:00 **Ons Rameh** (Université Paris-Cité & ENS-PSL)

Title: *Mixing time of the Asymmetric Zero Range process on the segment.*

- 15:00 - 15:30 coffee break

- 15:30 - 16:45 mini-course B (Part I), **Justin Salez** (Université Paris-Dauphine)

Title: *The cutoff phenomenon for finite Markov chains*

- 16:45 - 17:15 **Antoine Jégou** (EPFL, Lausanne)

Title: *Uniqueness of Malliavin—Kontsevich—Suhov measures*

- 17:15 - 17:45 **Faezeh Khodabandehlou** (KU Leuven)

Title: *Poisson equation for nonreversible Markov jump processes and its applications*

- 17 :45 – 18 :15 **Federico Sau** (Università degli Studi di Trieste)

Title: *Scaling limits of the averaging process*

Friday September 27 2024

- 09:00 – 10:15 mini-course B (Part II), **Justin Salez** (Université Paris-Dauphine)

Title: *The cutoff phenomenon for finite Markov chains*

- 10:15 - 10:45 **Francesco Pedrotti** (Université Paris-Dauphine)

Title: *Log-concavity along the Ornstein—Uhlenbeck flow and applications to Lipschitz transport maps*

- 10:45 - 11:15 coffee break

- 11:15 - 11:45 **Clement Erignoux** (INRIA, Lyon)

Title: *Cutoff for the Transience Time for the SSEP with Traps and the Facilitated Exclusion Process (FEP)*

- 11:45 - 12:15 **Brune Massoulié** (Université Paris-Dauphine)

Title: *Mixing time of the facilitated exclusion process (FEP) and the SSEP with traps,*

- 12:15 - 14:00 lunch

- 14:00 - 15:15 mini-course A (Part II), **Gunter Schütz** (Forschungszentrum Jülich)

Title: Duality and symmetry in interacting particle systems

-15:15 - 15:45 **Hong-Bin Chen** (IHES, Paris)

Title: *Free energy in non-convex mean-field spin glass models*

-15:45 - 16:15 **Anne-Laure Basdevant** (LPSM, Sorbonne Université)

Title: *Time constant in first passage percolation.*

Abstracts

Mini-courses

Gunter Schütz (Forschungszentrum Jülich)

Title: Duality and symmetry in interacting particle systems

Abstract: Duality is an important concept in the study of stochastic interacting particle systems. For arbitrary initial measures duality expresses expectations of a family of functions at time t in terms of the transition probability of a dual process which may be simpler to analyse. Focussing on countable state space we discuss duality from the perspective of the generator. Unlike the more traditional approach of looking at duality in a pathwise manner this allows us to understand straightforwardly how dualities arise from symmetries, or more generally, from invariant subspaces of the generator and leads to constructive methods for finding useful dualities. Also the new concept of reverse duality comes out naturally. It yields the full probability measure of the process at time t for a family of initial measures in terms of transition probabilities of the dual process and thus allows for the computation of arbitrary expectation values.

Justin Salez (Université Paris-Dauphine)

Title: *The cutoff phenomenon for finite Markov chains*

Abstract: The cutoff phenomenon is an abrupt transition from out of equilibrium to equilibrium undergone by certain Markov processes in the limit where the number of states tends to infinity. Discovered forty years ago in the context of card shuffling, it has since then been established in a variety of contexts, including random walks on graphs and groups, interacting particle systems, or MCMC algorithms. Nevertheless, a general theory is still missing, and identifying the general mechanisms underlying this mysterious phenomenon remains one of the most fundamental problems in the area of mixing times. In this talk, I will give a self-contained introduction to this fascinating question, and then present a recent approach based on entropy and curvature.

Talks

Anne-Laure Basdevant (LPSM, Sorbonne Université)

Title: *Time constant in first passage percolation*

Abstract: First passage percolation is a mathematical model of fluid propagation in an inhomogeneous medium. By subadditivity, there exists a constant giving the asymptotic speed of the fluid. However, very little is known about the value of this constant in the general case. In this talk, we look at such a model on \mathbb{R}^d , where the medium is almost homogeneous except in a few small areas of \mathbb{R}^d . We investigate the first order behaviour of the time constant as the size of the small inhomogeneities tends to 0. This talk is based on a joint work with Jean-Baptiste Gouéré

Hong-Bin Chen (IHES, Paris)

Title: *Free energy in non-convex mean-field spin glass models*

Abstract: We start by reviewing the classical Sherrington-Kirkpatrick (SK) model. In this model, ± 1 -valued spins interact with each other subject to random coupling constants. The covariance of the random interaction is quadratic in terms of spin overlaps. Parisi proposed the celebrated variational formula for the limit of free energy of the SK model in the 80s, which was later rigorously verified in the works by Guerra and Talagrand. This formula has been generalized in various settings, for instance, to vector-valued spins, by Panchenko. However, in these cases, the convexity of the interaction is crucial. In general, the limit of free energy in non-convex models is not known and we do not have variational formulas as valid candidates. Here, we report recent progress through the lens of the Hamilton-Jacobi equation. Under the assumption that the limit of free energy exists, we show that the value of the limit is prescribed by a characteristic line; and the limit (as a function) satisfies an infinite-dimensional Hamilton-Jacobi equation "almost everywhere". This talk is based on a joint work with Jean-Christophe Mourrat.

Clement Erignoux (INRIA, Lyon)

Title: *Cutoff for the Transience Time for the SSEP with Traps and the Facilitated Exclusion Process (FEP)*

Abstract: The facilitated exclusion process is a toy model for phase separation, where particles can jump to an empty neighboring site iff their other neighboring site is occupied. Because of this kinetic constraint, at low densities, the FEP ultimately reaches a frozen state where particles are all surrounded by empty sites, whereas at large densities, the FEP reaches an ergodic component where it can be mapped to the classical SSEP. In this talk, I will present a new mapping of the FEP to a process that we call SSEP with traps, which displays the same frozen/ergodic phases. I will then focus on the estimation on the transience time needed to reach either an ergodic or frozen state for this model on the ring, started from the "worst" possible state, which undergoes a cutoff as the size of the system diverges. Based on JW with Brune Massoulié (Université Paris Dauphine)

Antoine Jego (EPFL, Lausanne)

Title: *Uniqueness of Malliavin—Kontsevich—Suhov measures*

Abstract: About 20 years ago, Kontsevich & Suhov conjectured the existence and uniqueness of a family of measures on the set of Jordan curves, characterised by conformal invariance and a restriction-type property. This conjecture was motivated by (seemingly unrelated) works of Schramm, Lawler & Werner on Schramm—Loewner evolutions (SLE), and Malliavin, Airault & Thalmaier on "unitarising measures". The existence of this family was settled by works of Werner—Kempainen and Zhan, using a loop version of SLE. The uniqueness was recently obtained in a joint work with Baverez. I will start by reviewing the different notions involved before giving some ideas of our proof of uniqueness: in a nutshell, we construct a family of "orthogonal polynomials" which completely characterises the measure. I will discuss the broader context in which our construction fits, namely the conformal field theory associated with SLE.

Faezeh Khodabandehlou (KU Leuven)

Title: *Poisson equation for nonreversible Markov jump processes and its applications*

Abstract: We study the solution V of the Poisson equation $LV + f = 0$, where L is the backward generator of an irreducible (finite) Markov jump process and " f " is a given centred state function. We provide a graphical representation of the solutions of Poisson equations with different f . Bounds on V are obtained using a graphical representation derived from the Matrix Forest Theorem and

using a relation with mean first-passage times. Applications include estimating time-accumulated differences during relaxation toward a steady nonequilibrium regime.

The references are:

1. F. Khodabandehlou, C. Maes and K. Netocny. *On the Poisson equation for nonreversible Markov jump processes*, Journal of Mathematical Physics 65(4).(2024)
2. F. Khodabandehlou, C. Maes, I. Maes, and K. Netocny. *The vanishing of excess heat for nonequilibrium processes reaching zero ambient temperature*. Ann. H. Poincare, (2023).
3. F. Khodabandehlou, C. Maes, and K. Netocny . *Trees and forests for nonequilibrium purposes: An introduction to graphical representations*. J. Stat. Phys, 189:41, (2022).

Seonwoo Kim (KIAS, Séoul)

Title: *Metastable Hierarchy in Abstract Low-Temperature Lattice Models*

Abstract: The phenomenon of metastability, and especially its hierarchical decomposition, is ubiquitous in a large class of dynamical systems, both real-life and theoretic, with two or more locally stable states. In this talk, I will briefly review this phenomenon in the setting of abstract lattice models at low temperatures. I will also talk about a few examples which include Glauber and Kawasaki dynamics for Ising/Potts models. The talk is partially based on a joint work with Insuk Seo (SNU).

Brune Massoulié (Université Paris-Dauphine)

Title: *Mixing time of the facilitated exclusion process (FEP) and the SSEP with traps*

Abstract: The facilitated exclusion process (FEP) is a particle system, where particles evolve on a discrete lattice, making random jumps while obeying local constraints. Because of these constraints, it is nonreversible, and has transient configurations: after a finite time, it is no longer transient and is either frozen or reaches an ergodic component. The transience time has been studied in JW with Clément Erignoux (Inria), using a mapping to another process, the SSEP with traps. In the case where the FEP doesn't freeze and continues moving, an interesting question is the mixing time, ie the time such that the FEP's distribution becomes close to its invariant law. First, I will show cutoff for the SSEP with traps' mixing time, by showing the time spent in the transient phase and the ergodic phase balance each other out. Then, I will show cutoff for the FEP's mixing time, which is not a direct consequence of the latter but requires a more involved mapping.

Francesco Pedrotti (Université Paris-Dauphine)

Title: *Log-concavity along the Ornstein—Uhlenbeck flow and applications to Lipschitz transport maps*

Abstract: In this talk (based on joint work with Giovanni Brigati), we consider the problem of understanding whether an initial distribution evolved along the Ornstein—Uhlenbeck flow eventually becomes log-concave. This is a natural question, since the (standard) Gaussian limiting distribution is strongly log-concave.

We show that assuming only subgaussianity of the initial measure is not enough for the conclusion to hold. On the other hand, we identify a class of distributions for which the answer is positive.

As a corollary, we derive new estimates for the Lipschitz constant of a transport map constructed by Kim and Milman, with applications to functional inequalities for the aforementioned measures.

Ons Rameh (Université Paris-Cité & ENS-PSL)

Title: *Mixing time of the Asymmetric Zero Range process on the segment.*

Abstract: In this talk, we shall discuss the mixing time of the Asymmetric Zero Range process (AZRP) with non-decreasing jump rates on the segment. We will first study the hydrodynamic limit starting from the configuration where all particles are gathered on the left-most site. The macroscopic equilibrium time gives a lower bound on the mixing time. We will show the sharpness of this estimate when the system is asymmetric enough, which establishes the cut-off phenomenon. This extends earlier results of Labbe and Lacoïn in the context of the asymmetric simple exclusion process.

Federico Sau (Università degli Studi di Trieste)

Title: *Scaling limits of the averaging process*

Abstract: The averaging process on a graph is a continuous-space Markov chain, which is commonly interpreted as an opinion dynamics, a distributed algorithm, or an interface moving through a randomized sequence of deterministic local updates. Its dynamics goes as follows. Attach i.i.d. Poisson clocks to edges, and assign real values to vertices; at the arrival times of these clocks, update the values with their average. As time runs, the averaging process converges to a flat configuration, and one major problem is that of quantifying the speed of convergence to its degenerate equilibrium.

In this talk, after reviewing some basic properties and recent results on mixing times for the averaging process on general graphs, we then focus on the discrete d -dimensional torus, discussing some quantitative features (e.g., limit profile, early concentration and local smoothness), and looking at nonequilibrium fluctuations.