On the generalized Burger equation

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Abstract. The present talk consists of two parts.

In the first part we consider the boundary-value problems as well as the Cauchy problem for one dimensional equation

$$u_t + g(t,u)u_x = \varepsilon u_{xx} - f(t,u).$$

Here $-f(t,u)$ plays the role of the absorption, the simplest case $f = \lambda u$, λ is positive constant. We formulate conditions guaranteeing the a priori estimate of $\max |u_x|$ independent of ε and t and give an example demonstrating the optimality if this condition. Based on this estimate we prove the global solvability of the above problems.

The second part is devoted to the Dirichlet problem for the multidimensional equation

$$u_t + \mathbf{g}(t,\mathbf{x},u) \cdot \nabla u = \varepsilon \Delta u + f(u).$$

Here $f(u)$ is a (nonlinear) source, for example $f(u) = u^p$, $p \geq 1$ or $f(u) = e^u$. It is well known that generally the solution of the Dirichlet problem for the above equation may blow-up in finite time, i.e.

$$\lim_{t \to t*} \max_x |u(t,\mathbf{x})| = +\infty$$

for some $t^* < +\infty$. We show that under the certain conditions the convective term

$$\mathbf{g}(t,\mathbf{x},u) \cdot \nabla u$$

guarantees the boundedness of a solution for any $t > 0$ and give the example demonstrating the optimality of this condition. Based on this we prove the global solvability of the Dirichlet problem.